



Tales AR

Artırılmış Gerçeklik Geometri Kitabı



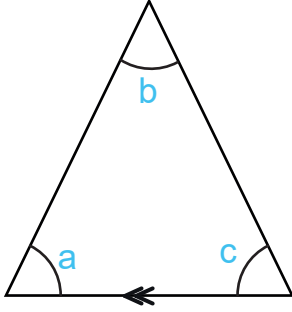
Uygulamayı Ücretsiz İndir!

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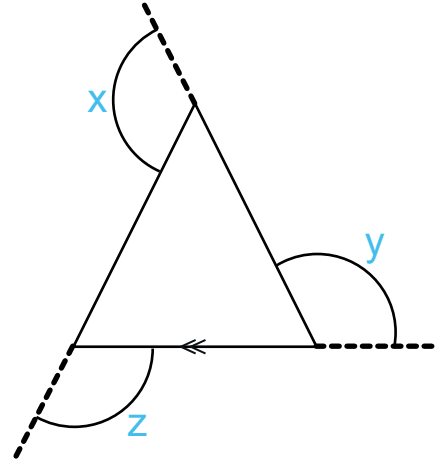
Üçgenin İç Açıları



İki Z kuralı ile
 $a + b + c = 180$ derecedir.



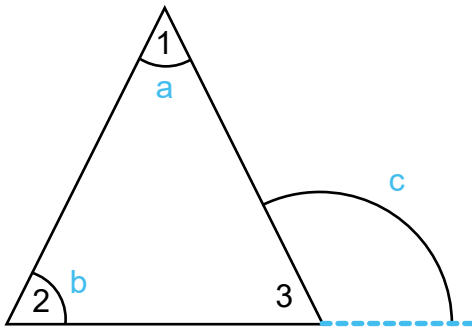
Üçgenin Dış Açıları



$x + y + z = 360$ derecedir.

TalesAR

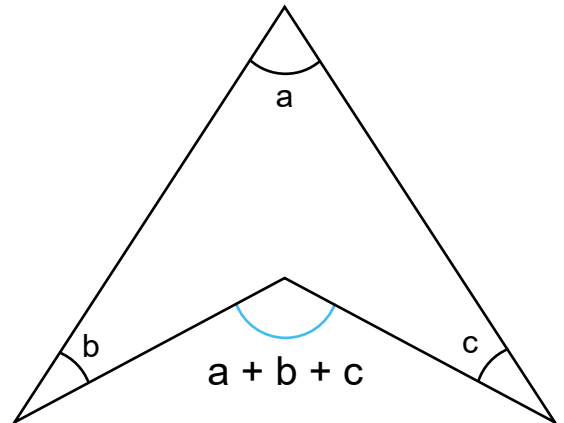
İki İç Açı, Bir Dış Açıya Eşittir.



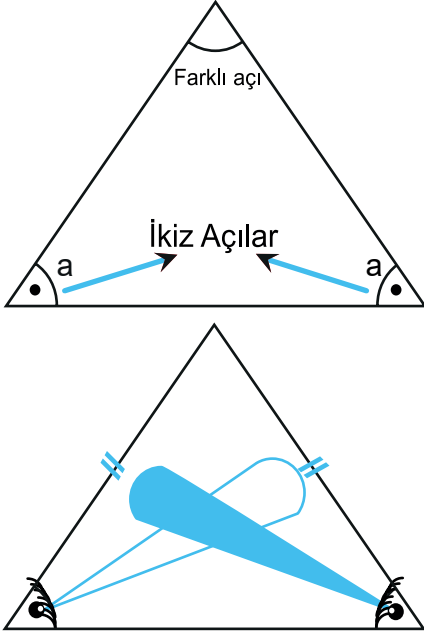
İki iç açının toplamı (1 ve 2), bilinmeyen 3. açığı 180'e tamamlayan kısma eşittir.

$$a + b = c$$

Bumerank

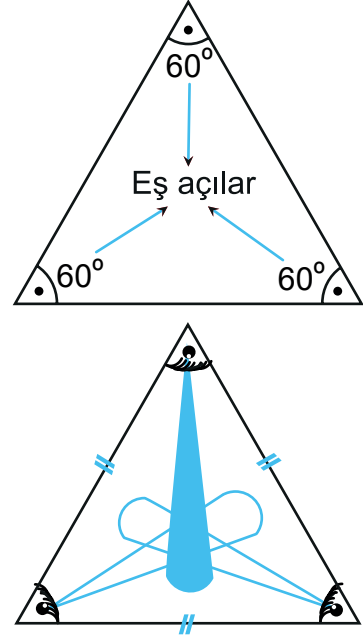


İkizkenar Üçgen



İkiz olan açılar göz gibi düşünürsek, gözler yani açılar birbirlerine eşit iseler gördükleri kenarlarda bir birlerine eşittirler.

Eşkenar Üçgen

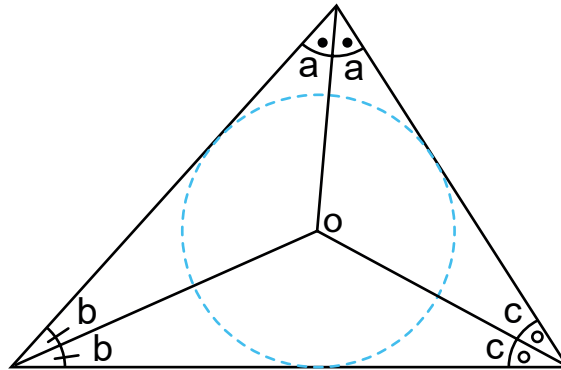


Eşkenar üçgende açılar göz gibi düşünürsek gördükleri kenarlar birbirine eşittir. Bu nedenle üçgende açılar birbirlerine eşittir ve 60 derecedir.

TalesAR

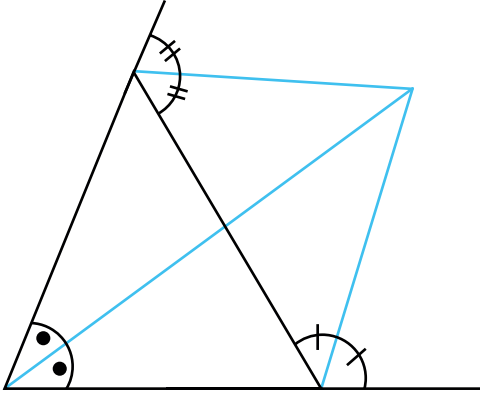


İç Teğet Çember

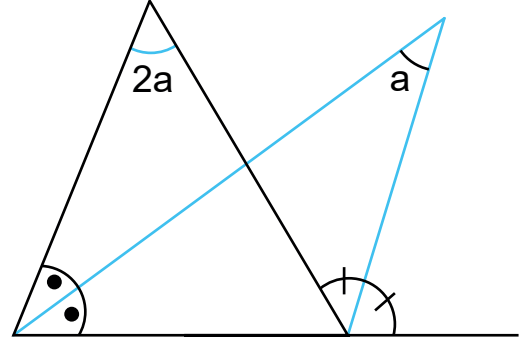




Dış Açortay

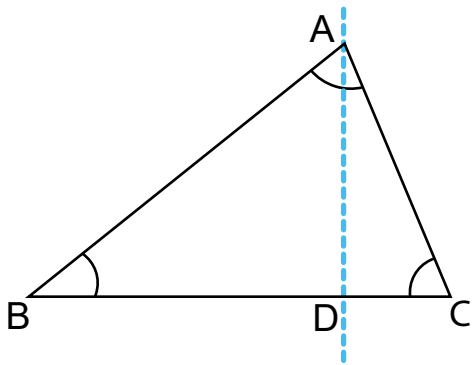


Dış Teğet Çember



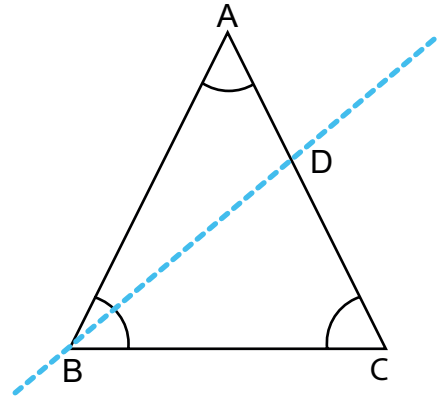
TalesAR

Nokta Doğru Üzerine Katlama



C noktası [BD] doğrusu üzerine katlanıyor

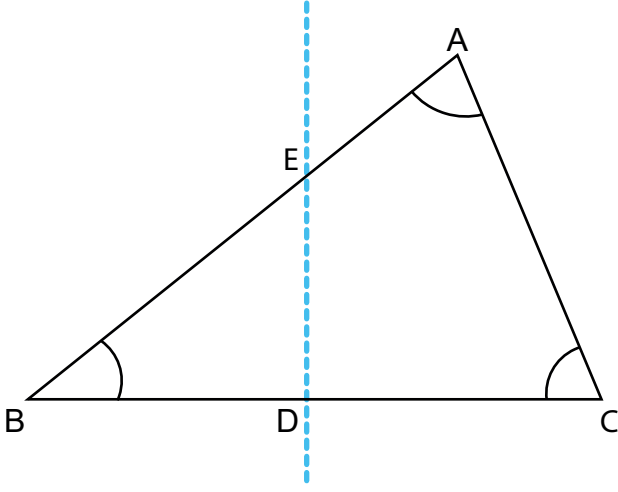
Köşeden Geçen Katlama



[BD] doğrusu boyunca katlanıyor.

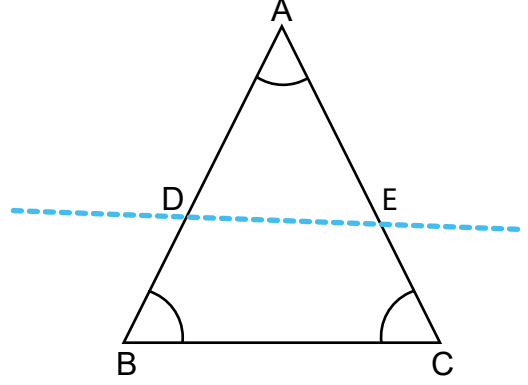


Nokta Nokta Üzerine



B noktası C noktası üzerine gelecek şekilde [DE] doğrusu boyunca katlanıyor.

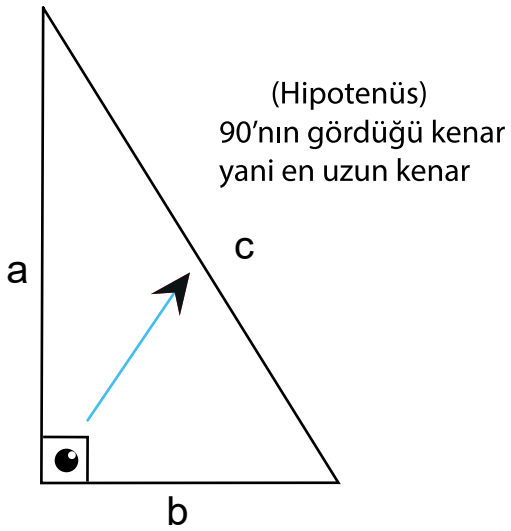
Rastgele Doğru Katlaması



[DE] doğrusu boyunca katlanıyor.

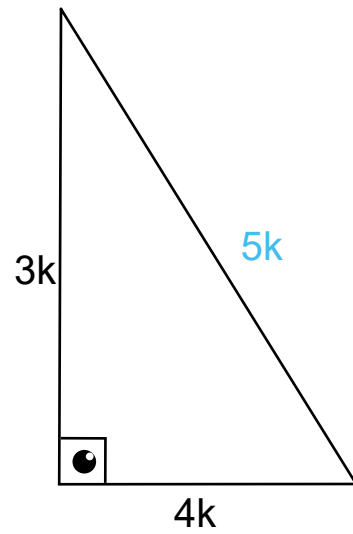
TalesAR

Pisagor

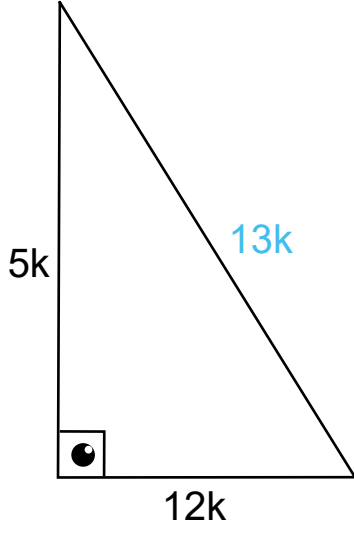


$$a^2 + b^2 = c^2$$

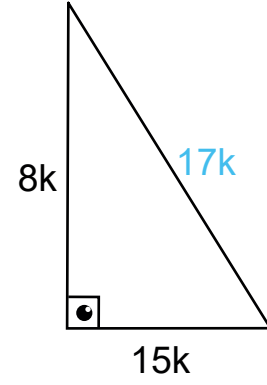
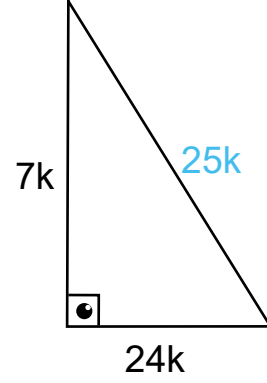
3,4,5 Özel Dik Üçgeni



5,12,13 Özel Dik Üçgeni



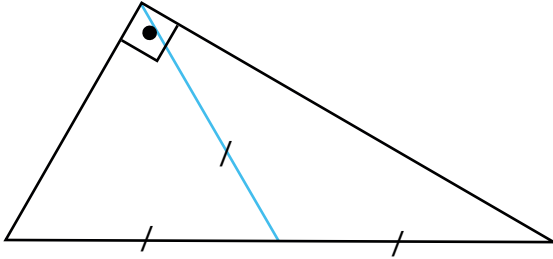
Özel Dik Üçgenler



TalesAR

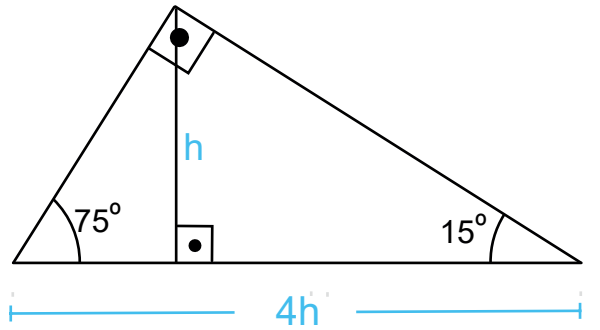


Muhteşem Üçlü



Hipotenüse inen kenarortay muhteşem üçlüyü oluşturur.

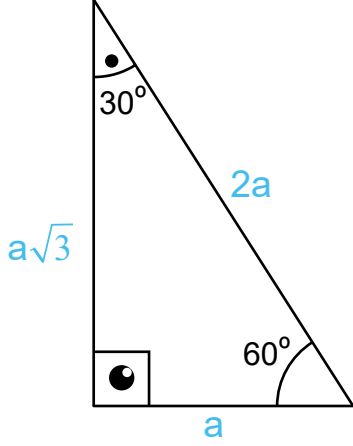
15 - 75 - 90 Üçgeni



15 , 75 , 90 üçgenindeki yüksekliğin 4 katı hipotenüse eşittir.

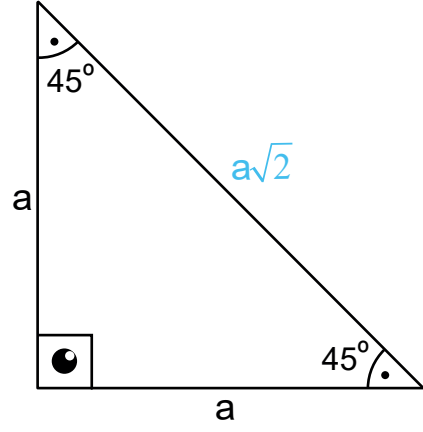


30 - 60 - 90 Üçgeni



30 dereceyi baz alıyoruz.
60 derecenin karşısı 30 derecenin $\sqrt{3}$ katı,
90 derecenin karşısı 30 derecenin 2 katıdır.

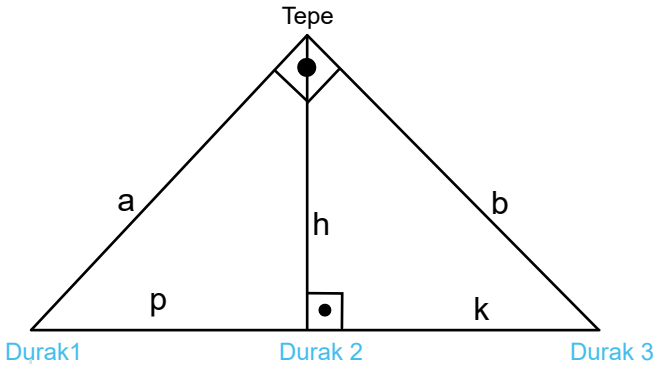
45 - 45 - 90 Üçgeni



İkizkenar dik üçgende 45 derecelerin karşıları birbirine eşit , 90 derecenin karşısı $\sqrt{2}$ katına eşittir.

TalesAR

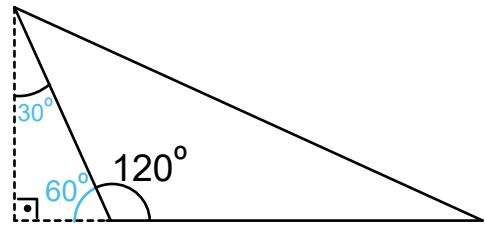
Dikten Dik Öklid



$$h^2 = p \cdot k$$
$$a^2 = p \cdot (p + k)$$
$$b^2 = p \cdot (k + p)$$

Tepeden herhangi bir yolu kullanmak için kare alınmalıdır. İnilen duraktan diğer duraklara uğranmalı ve bu yollar birbirleriyle çarpılmalıdır.

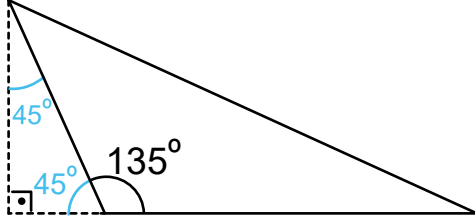
Geniş Açılı Üçgenlerdeki Dik Üçgenler



Dik üçgen çift ya da köklü olan kenardan uzatılmalıdır.

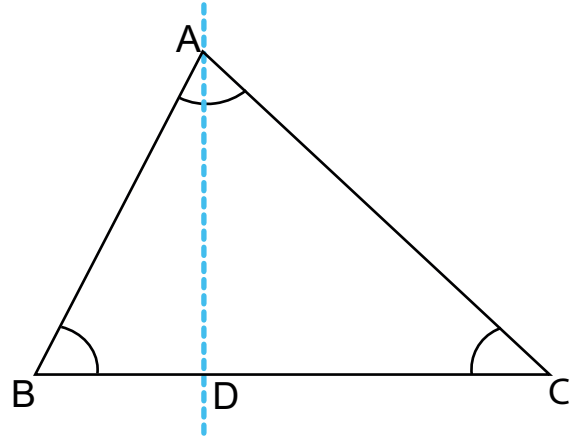


Geniş Açılı Üçgenlerdeki Dik Üçgenler



Dik üçgen çift ya da köklü olan kenardan uzatılmalıdır.

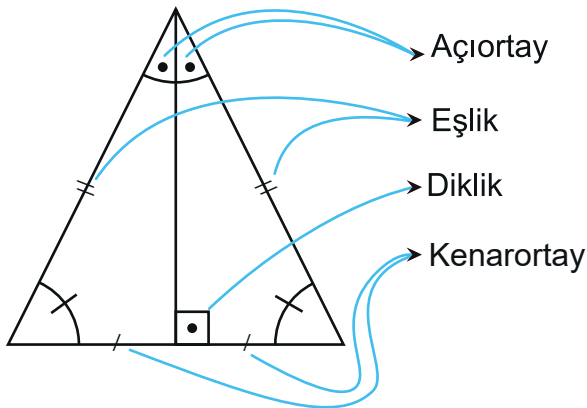
Nokta Doğru Üzerine Katlama



B noktası [BC] doğrusu üzerine katlanıyor. Nokta, olduğu doğru üzerine katlanabiliyorsa her zaman dik üçgen oluşur.

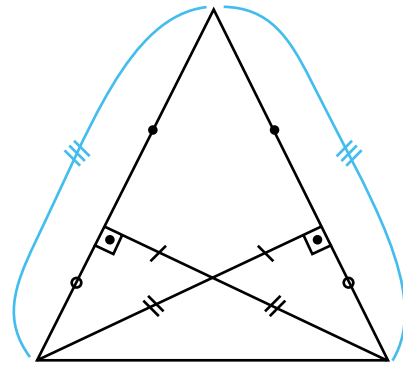
TalesAR

İkizkenar Üçgen



Bir üçgende eğer ki 2 tanesi var ise geri kalanlarda vardır.

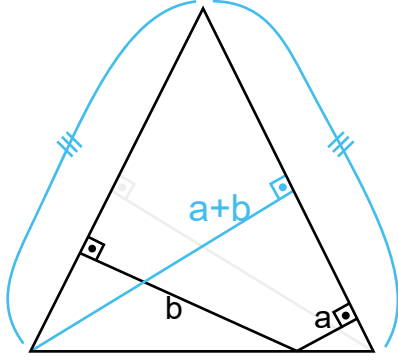
İkizkenarlara Dikme



İkizkenar üçgende eşit olan kenarlara inilen dikmeler birbirlerine eşittir. Ayrıca böldükleri parçalar da yukarıdaki gibi eşittir.

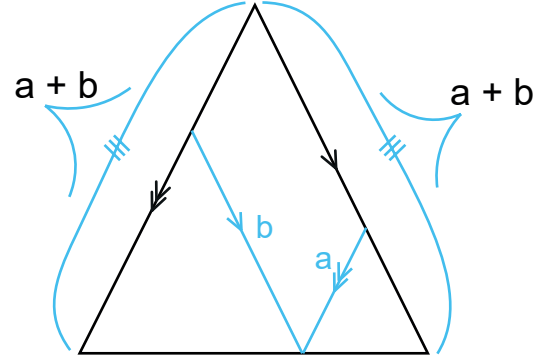


İkizkenarlara Dikmeler



Eşit olmayan kenardan yani tabandan alınan herhangi bir noktadan ikiz olan kenarlara çekilen dikmelerin toplamı ikizkenar üçgenin yüksekliğine eşittir.

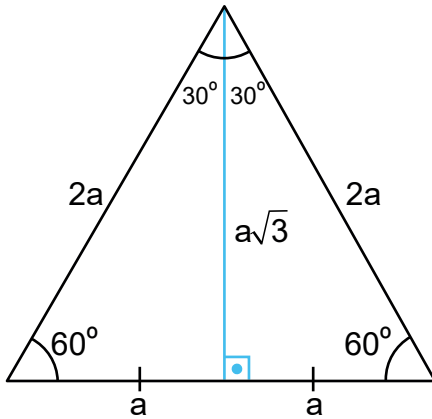
İkizkenarlara Paralel



Eşit olmayan kenardan yani tabandan alınan herhangi bir noktadan ikiz olan kenarlara çekilen paralellerin toplamı ikiz olan kenarlara eşittir.

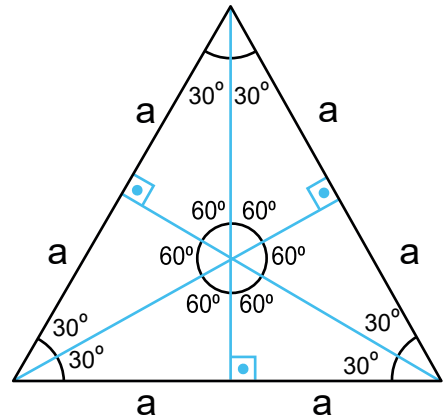
TalesAR

Eşkenar Üçgen



Eşkenar üçgen ikizkenarın bütün özelliklerini içerir. Herhangi bir yerden inilen dikme 30 60 90 üçgenini oluşturur.

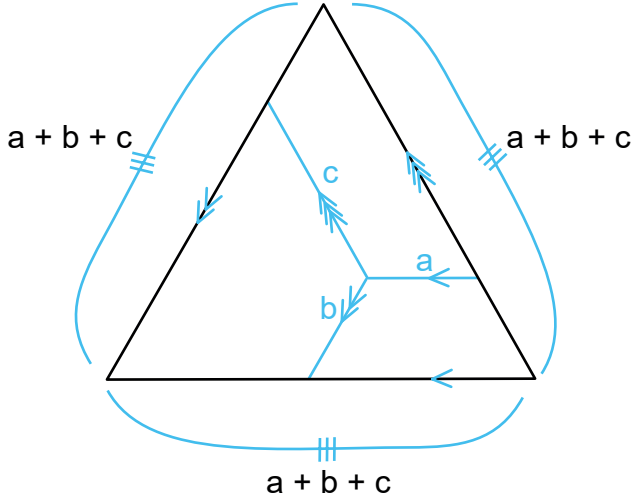
Eşkenar Üçgen



Bütün yükseklikler aynı olduğu için eşit 30 , 60 , 90 üçgenleri oluşur.

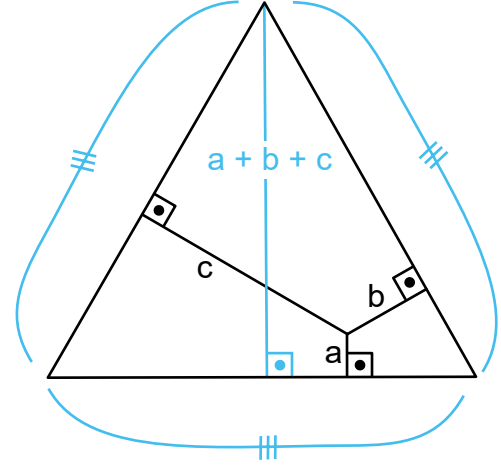


Eşkenarlara Paralel



Bu üç paralelin toplamı eşkenar üçgenin bir kenarına eşit olur.

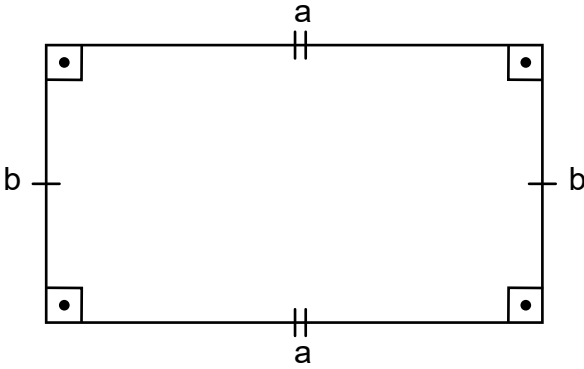
Eşkenarlara Dikme



Eşkenar üçgende her kenar birbirine eş olduğu için içeriden bir nokta alır ve eş olan kenarlara dikme ineriz. Bu üç dikmenin toplamı eşkenar üçgenin yüksekliğine eşittir. İlk modelde gördüğümüz gibi bütün kenarlardan çekilen yükseklikler birbirlerine eşittir.

TalesAR

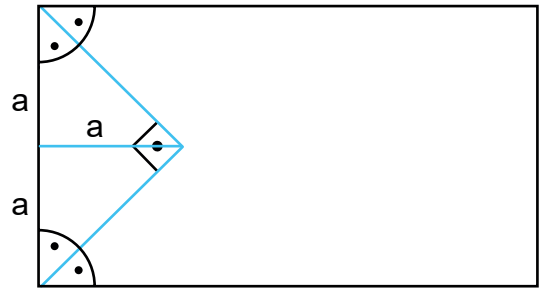
Dikdörtgen Çevre ve Alan



$$\text{Çevre} = 2a + 2b$$

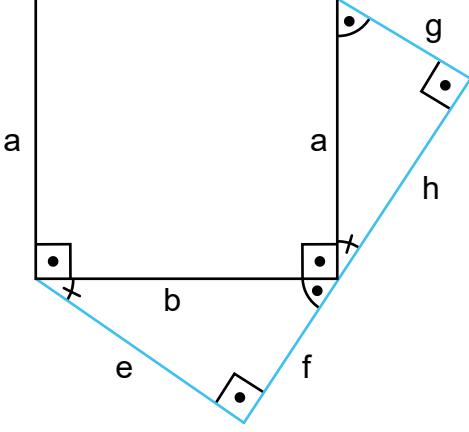
$$\text{Alan} = a \cdot b$$

Dikdörtgen Muhteşem 3'lü



Dikdörtgende açıortayların kesim noktası ikizkenar muhteşem üçlüyü oluşturur.

Dikdörtgen Benzerlik Üçgeni

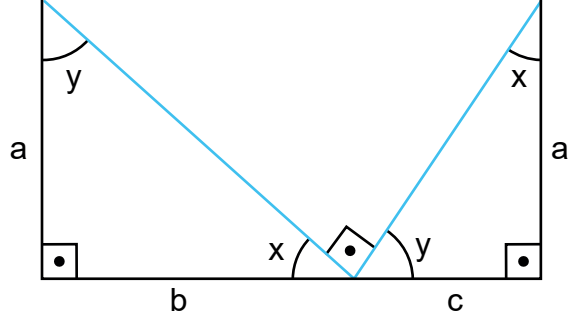


$$\frac{b}{a} = \frac{e}{h} = \frac{f}{h}$$



TalesAR

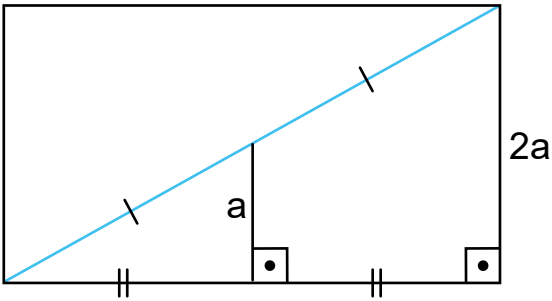
Dikdörtgen Benzerlik Üçgeni



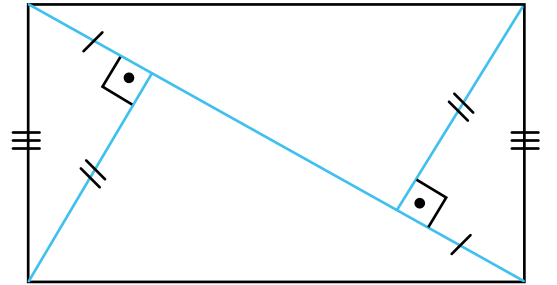
$$\frac{a}{c} = \frac{b}{a}$$



Dikdörtgen Orta Taban

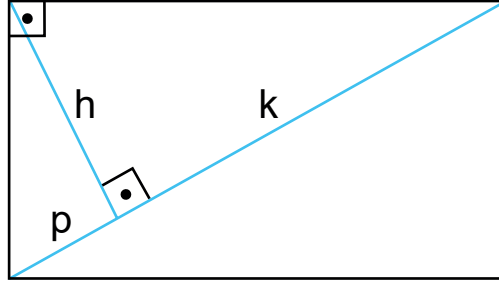


Dikdörtgen Eş Üçgenler



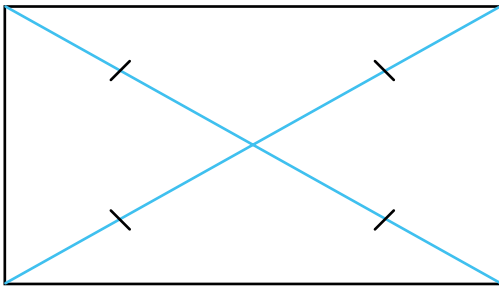
Köşegene inen eş dikmeler birbirine eş iki üçgen oluşturur.

Dikdörtgen Oklid

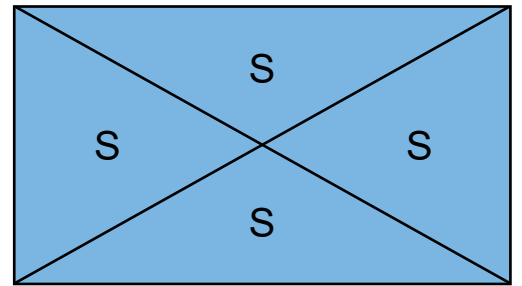


TalesAR

Dikdörtgen Köşegenler



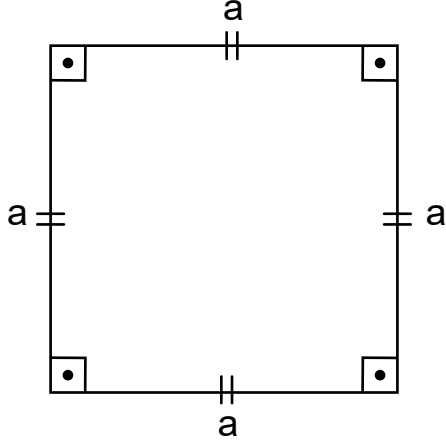
Dikdörtgen Bölünmüş Alan



Dikdörtgende köşegenler birbirlerini ortalarlar.



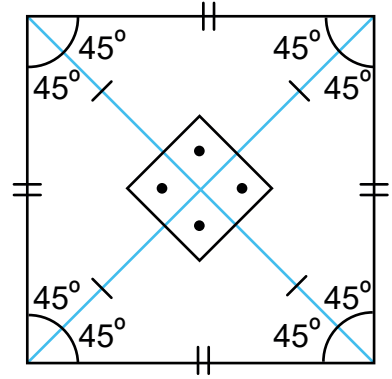
Kare Alan ve Çevre



$$\text{Çevre} = 4a$$

$$\text{Alan} = a^2$$

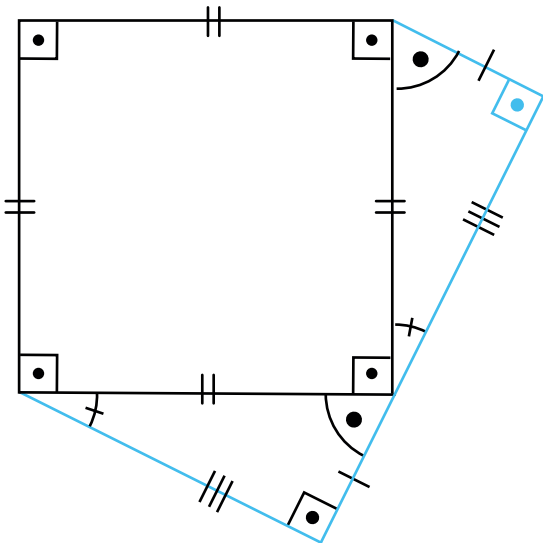
Kare Köşegenler



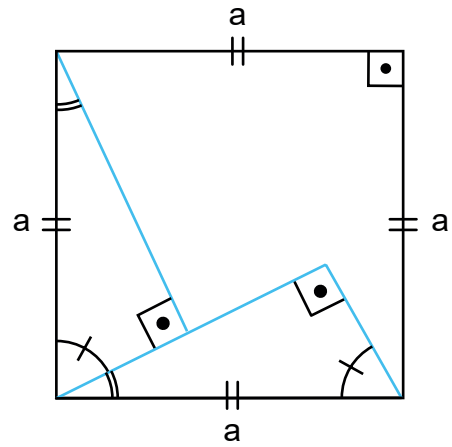
Karede köşegenler, birbirlerini dik keser ve eş dört üçgene böler.

TalesAR

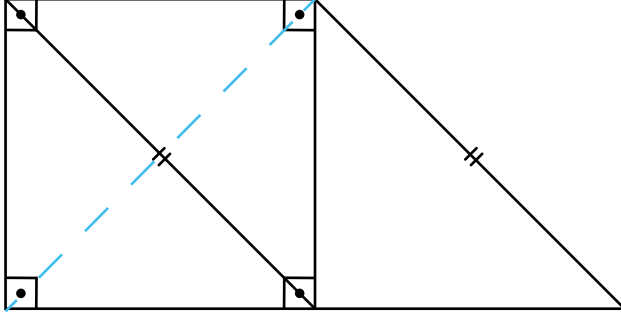
Kare Dışı Eş Üçgenler



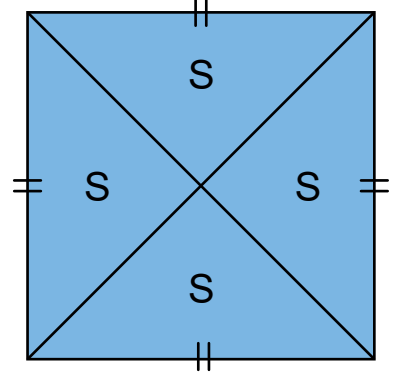
Kare İçi Eş Üçgenler



Kare Eş Köşegenler



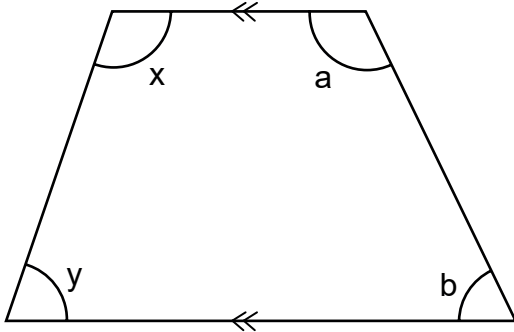
Kare Bölünmüş Alanlar



Köşegenlerin birbirlerine eş oldukları için verilen eşitlikler ile ikizkenar üçgenler bulunabilir animasyonda anlatılmıştır.

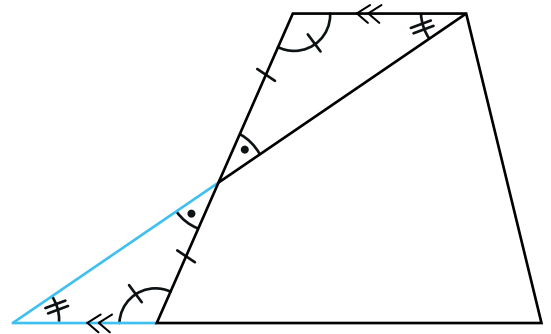
TalesAR

Yamuk Açıları



$$x + y = 180^\circ$$
$$a + b = 180^\circ$$

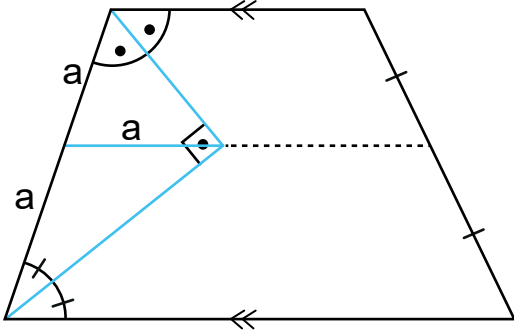
Yamuk Kelebek



Yamukta aşağıya ulaşamayan doğruları uzatırsanız kelebek olacaktır. Eğer orta noktadan uzatırsanız eş üçgenler oluşur.

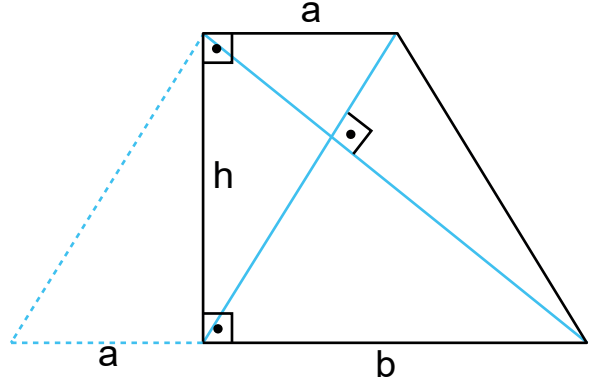


Yamuk Muhteşem 3'lü



Yamukta açkırtaylar orta tabanın üstünde keşirler.

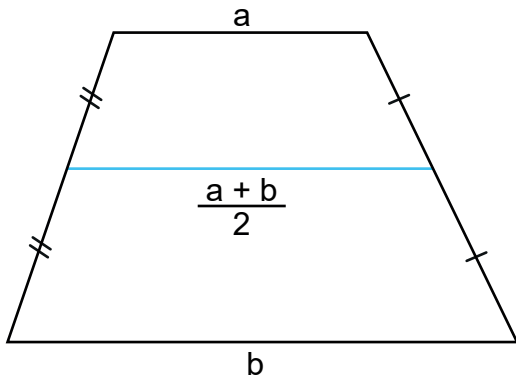
Yamuk Oklid



$$h^2 = a \cdot b$$

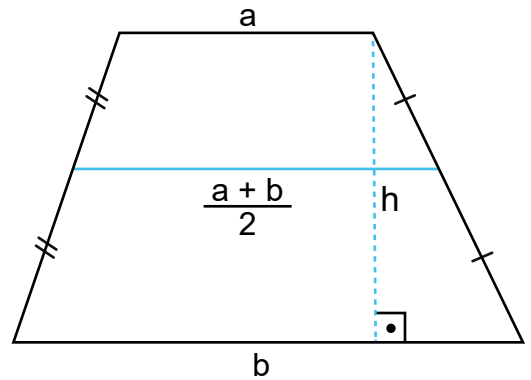
TalesAR

Yamuk Orta Taban



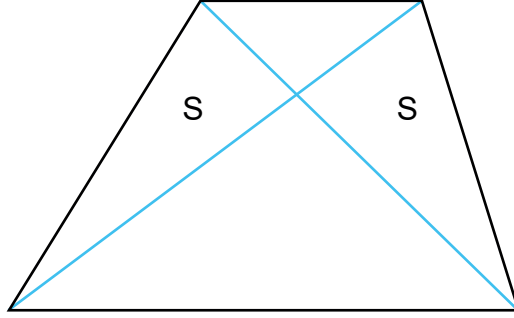
Orta taban üst taban ile alt tabanın toplamının yarısıdır.

Yamuk Alan



$$\text{Alan} = \frac{a+b}{2} \cdot h$$

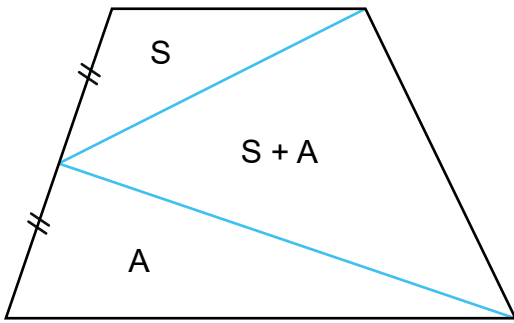
Yamuk Kanat Alanları Eşit



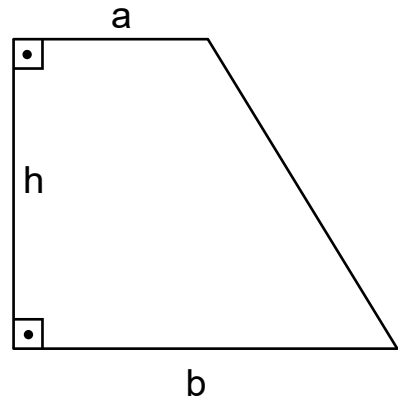
Şekilde görüldüğü gibi kanat dediğimiz alanlar birbirlerine eşittir.

TalesAR

Yamuk Alanlar Toplamı



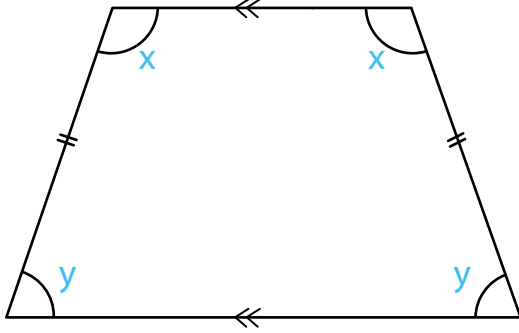
Dik Yamuk



Eğer ki orta noktadan köşelere doğrular çekilirse şekildeki gibi olacaktır.

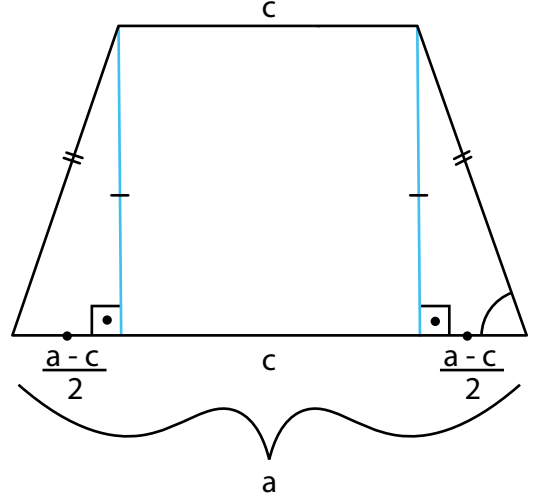


İkizkenar Yamuk



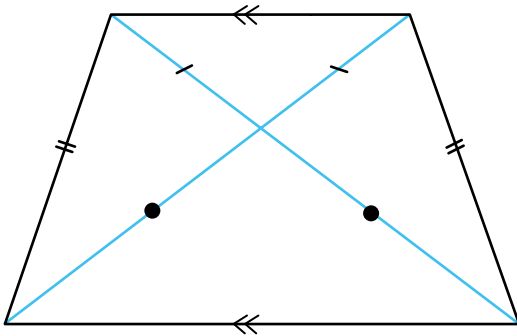
İkizkenar yamuk ikizlikten dolayı açılar eşittir.

İkizkenar Yamuk Eş Dik Üçgenler

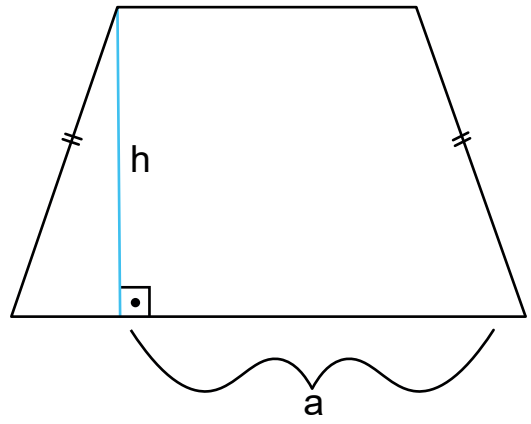


TalesAR

İkizkenar Yamuk Eş Üçgenler



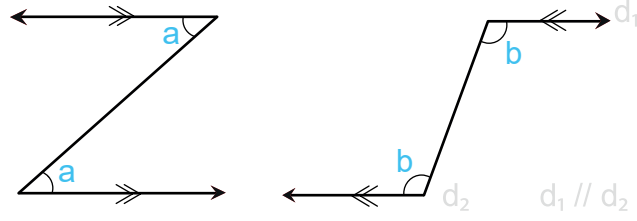
İkizkenar Yamuk Alan



$$\text{Alan} = h \cdot a$$



Z Kuralı

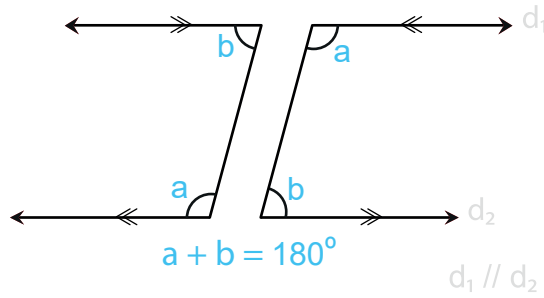


Paralel bir doğrudan başlayarak, tek bir kırılım ile aşağıdaki başka bir paralel doğruya ulaşan ve ters yönde devam eden doğrudur.

TalesAR



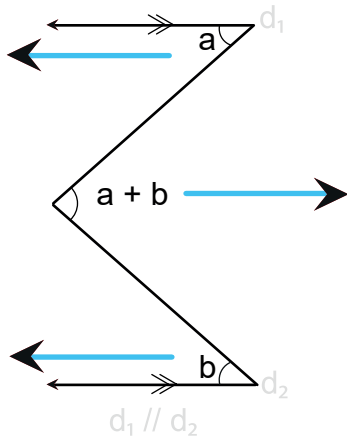
U Kuralı



Paralel bir doğrudan başlayarak, tek bir kırılım ile aşağıdaki başka bir paralel doğruya ulaşan ve aynı yönde devam eden doğrudur.

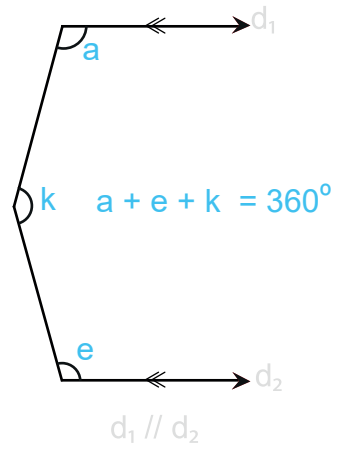


M Kuralı (iki z kuralı)



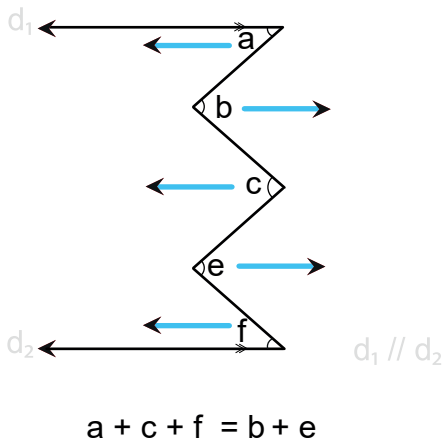
Sola bakan açıların toplamı , sağa bakan açiya eşittir.

Kalem Ucu (iki u kuralı)



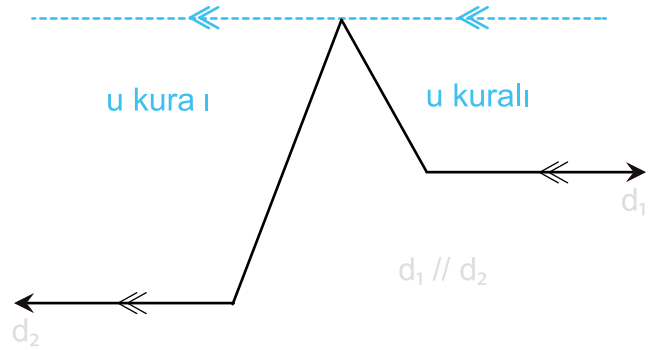
TalesAR

Zikzak Kuralı



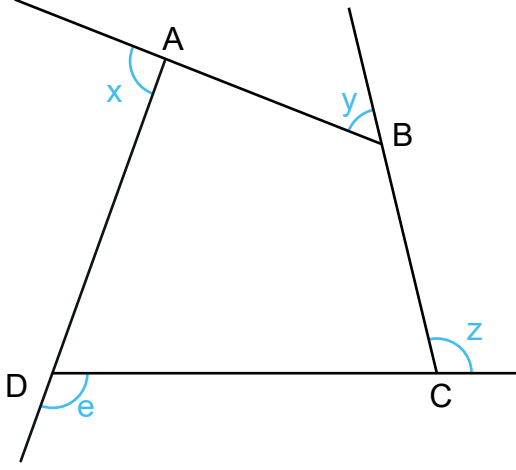
Bir paralel doğrudan başlayarak diğer bir paralel doğruya ulaşana kadar sağa bakan açıdan sonra sola bakan bir açıyı seçecek şekilde uygulanan kuraldır.

Sivri Noktadan Paralel



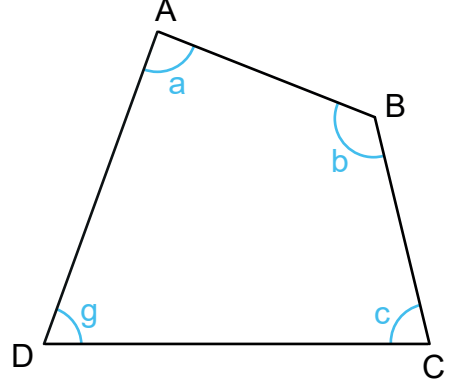


Dörtgende Dış Açılar



$$x + y + z + e = 360 \text{ derecedir.}$$

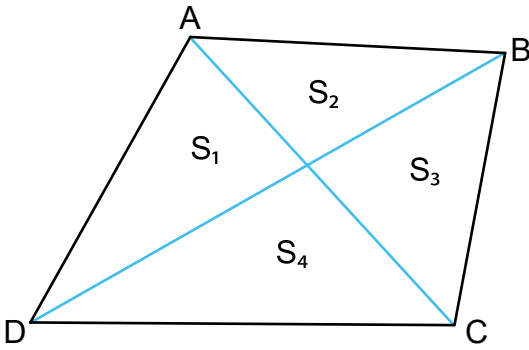
Dörtgende İç Açılar Toplamı



$$a + b + c + g = 360 \text{ derecedir.}$$

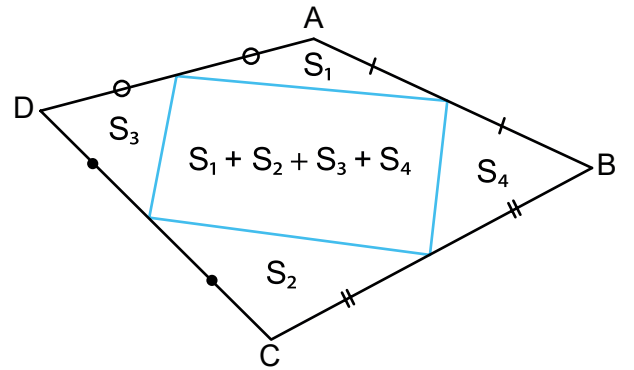
TalesAR

Dörtgen Alanlar Çarpımı



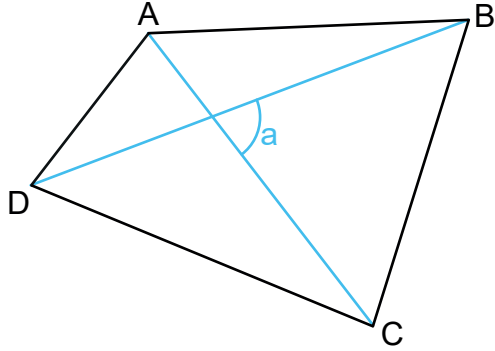
$$S_1 \cdot S_3 = S_2 \cdot S_4$$

Alanlar Ortaya Eşit



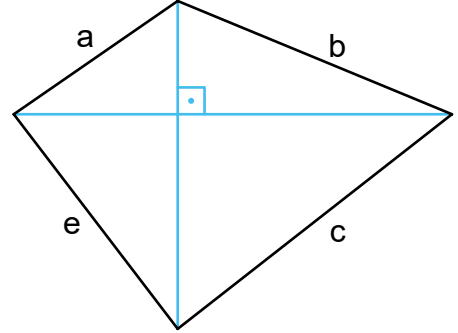
$$S_1 + S_2 = S_3 + S_4$$

Bütün Dörtgende Alan



$$\text{Alan}(ABCD) = \frac{|AC| \cdot |DB|}{2} \cdot \sin a$$

Dörtgen Dik Üçgen Kuralı

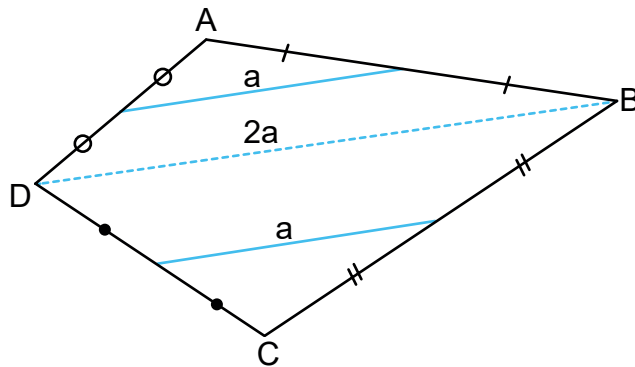


$$a^2 + c^2 = e^2 + b^2$$

TalesAR

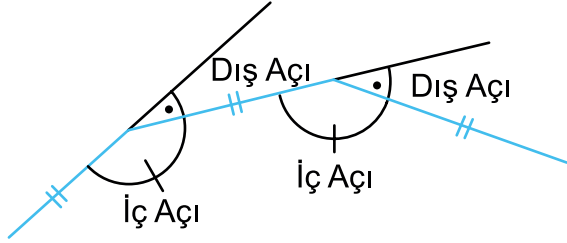


Dörtgenler Orta Taban



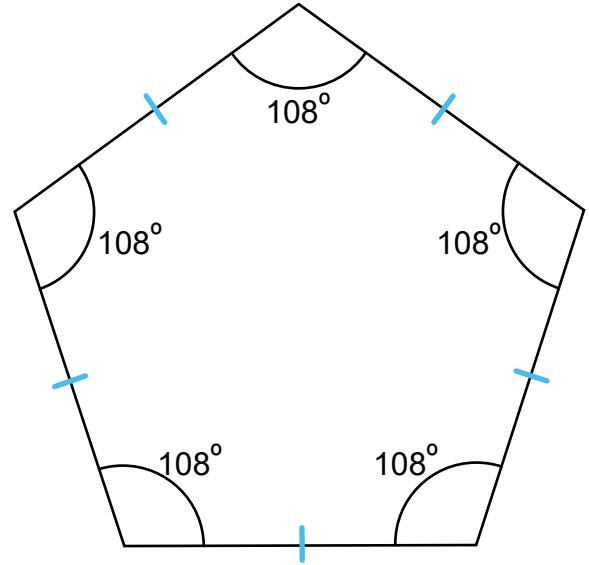


Çokgenler İç ve Dış Aç



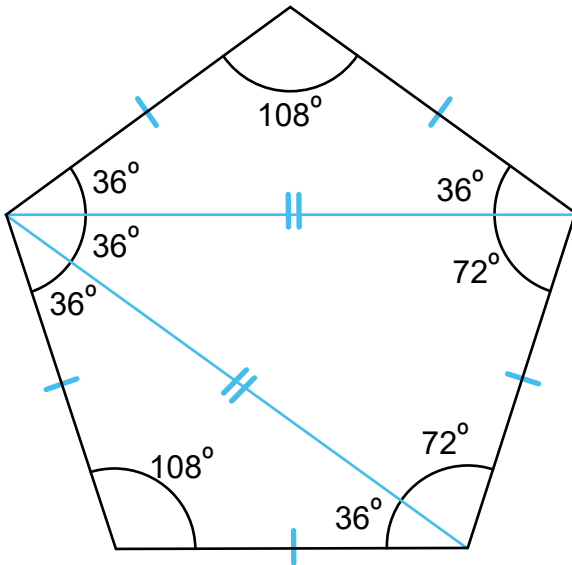
n kenarlı bir düzgün çokgenin iç açılar toplamı $= (n - 2) \cdot 180$

Beşgen

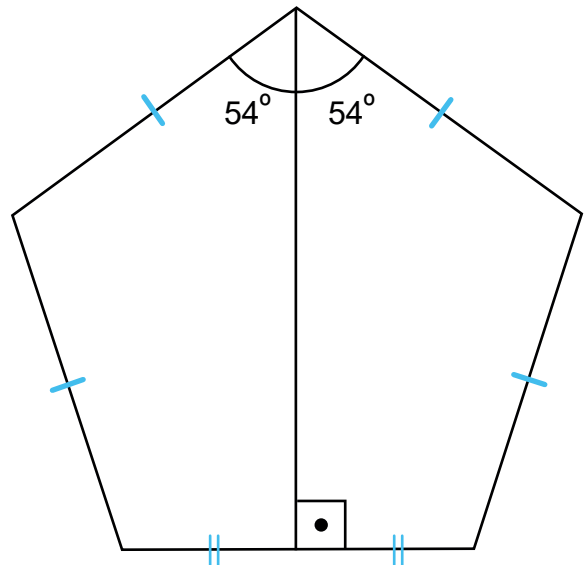


TalesAR

Beşgen Köşegenleri



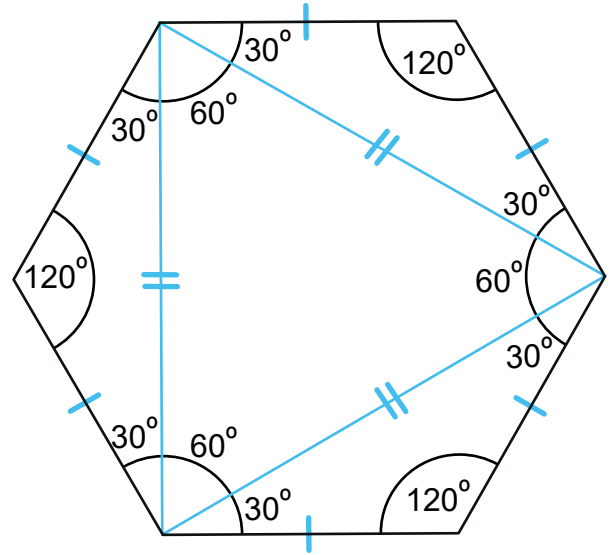
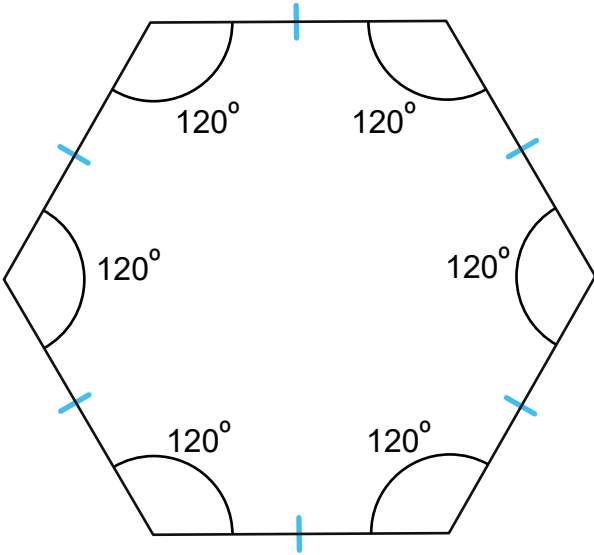
Beşgen Simetri Doğrusu





Altıgen

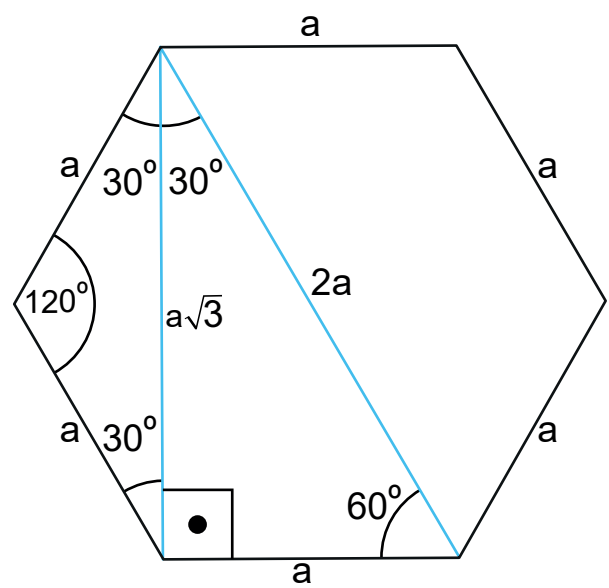
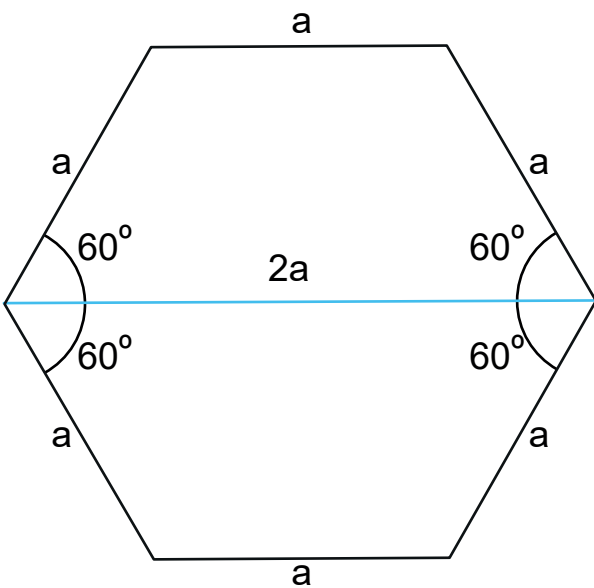
Altıgen Kısa Köşegenleri



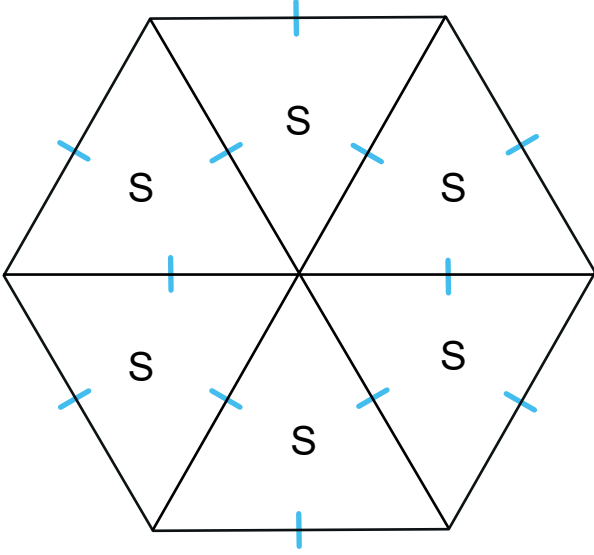
TalesAR

Altıgen Uzun Köşegen

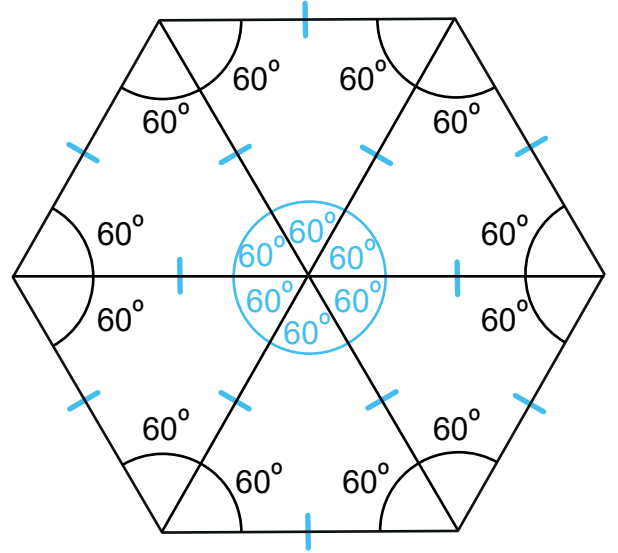
Altıgen Köşegenler Üçgeni



Altıgen Eş Alanlar

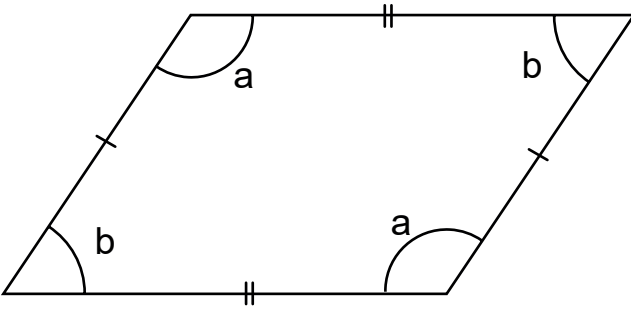


Altıgen Eşkenar Üçgenleri



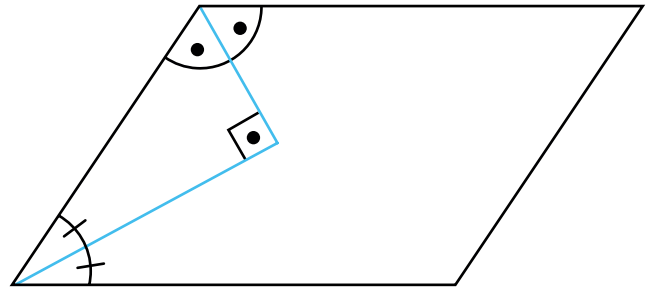
TalesAR

Paralelkenar



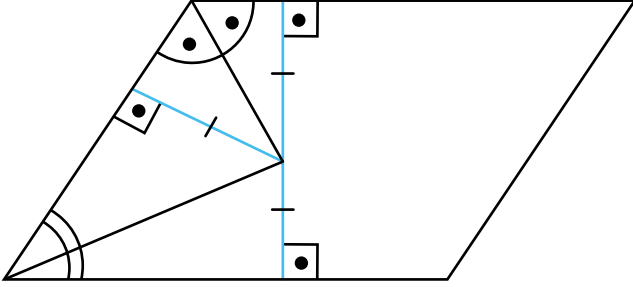
Paralelkenarda karşılıklı açılar ve uzunluklar birbirlerine eşittir.

Paralelkenar Açıortaylar



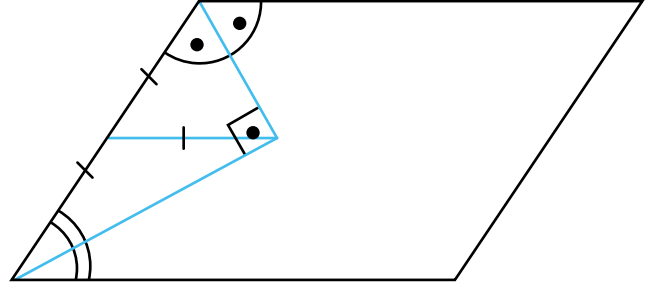
Paralelkenarda açıortayların kesiştikleri nokta doksan derecedir.

Paralelkenar Deltiod



Paralelkenarda deltoidler kullanılarak yükseklik bulunabilir.

Paralelkenar Muhteşem 3'lü

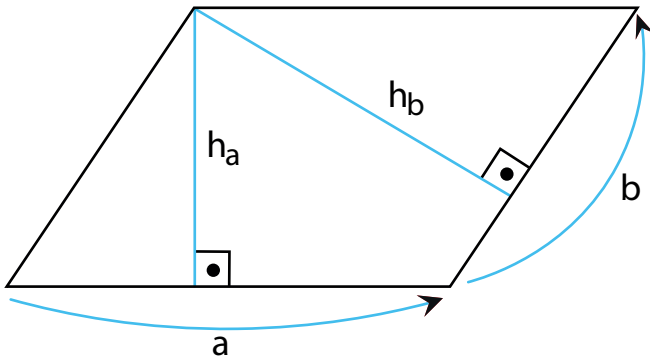


Paralelkenarda açıortayların kesiştikleri nokta doksan derecedir ve muhteşem 3'lü oluşturur.

TalesAR



Paralelkenar Alan



Alan : Taban . Yükseklik

$$\text{Alan} : h_a \cdot a = h_b \cdot b$$

Paralelkenar Alan

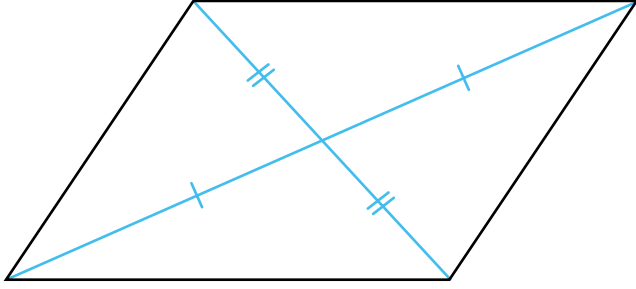


Paralelkenar animasyondaki gibi bir dikdörtgenin alanına eşittir.

$$\text{Alan} = \text{Taban} \cdot h$$

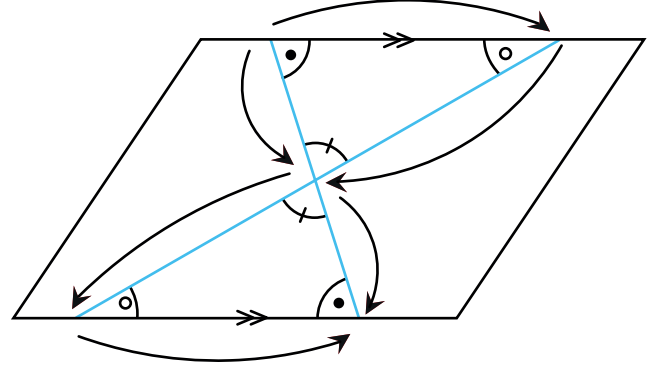


Paralelkenar Köşegenler



Paralelkenarda köşegenler birbirlerini ortalar.

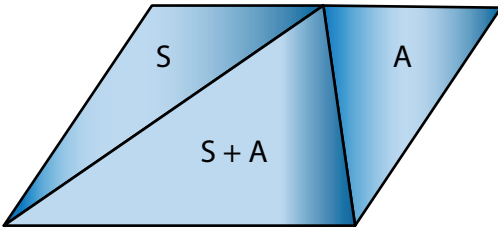
Paralelkenar Kelebek



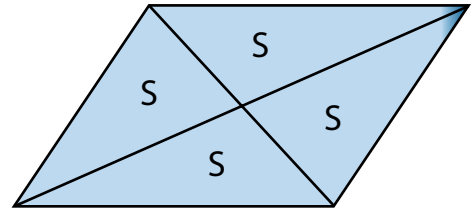
Paralelkenarda üst kenar alt kenara paralel olduğu için kelebek yapılabilir.

TalesAR

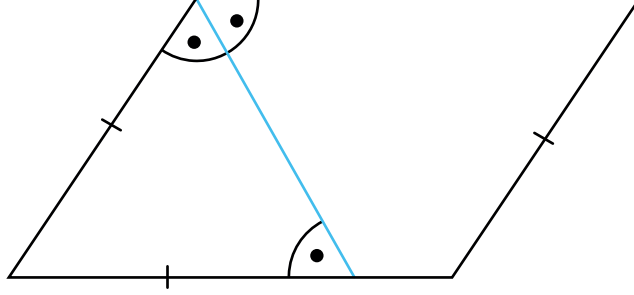
Paralelkenar Toplamı



Paralelkenar Alan



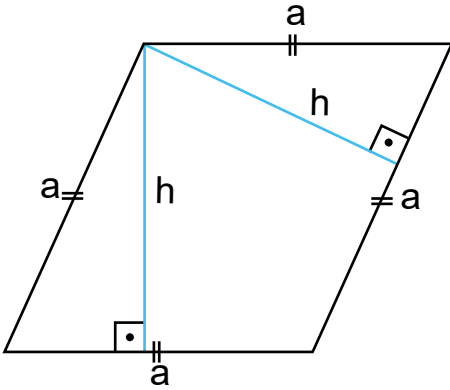
Paralelkenar Z Kuralı



Paralelkenarda z kuralını kullanarak ikizkenar üçgenler bulabiliriz.

TalesAR

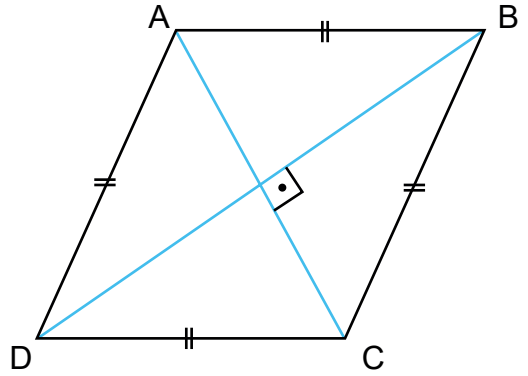
Eşkenar Dörtgen
Alan ve Çevre



$$\text{Çevre} = 4a$$

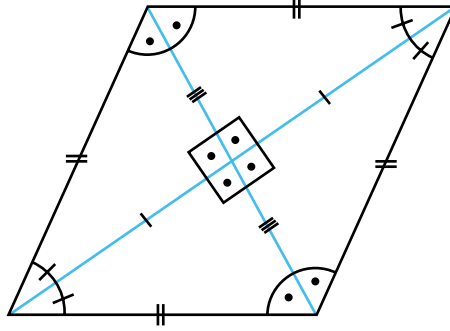
$$\text{Alan} = a \cdot h$$

Eşkenar Dörtgen
Köşegenlerden Alan



$$\text{Alan} = \frac{|AC| \cdot |BD|}{2}$$

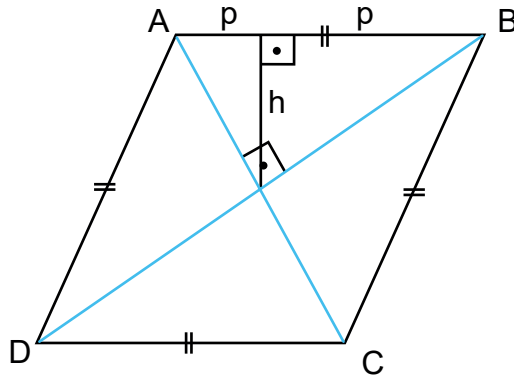
Eşkenar Dörtgen Köşegenler



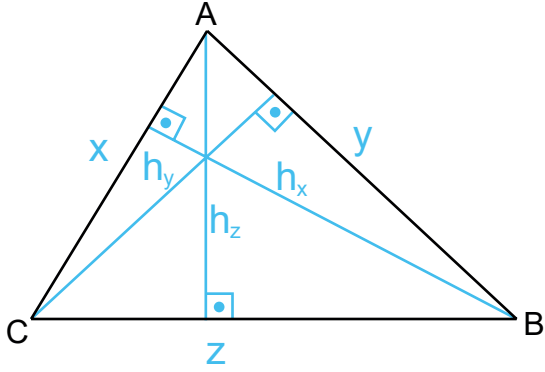
Eşkenar Dörtgende köşegenler birbirlerini ortalar ve eş dört dik üçgene ayırır.

TalesAR

Eşkenar Dörtgen Oklid



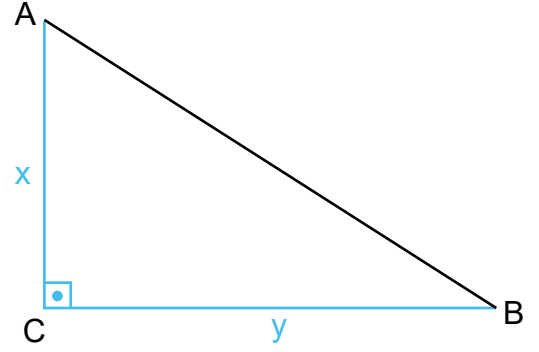
Dar Açılı Üçgende Alan



$$\text{Alan}(\triangle ABC) = \frac{x \cdot h_x}{2} = \frac{y \cdot h_y}{2} = \frac{z \cdot h_z}{2}$$

Dar açılı üçgenlerde diklik merkezi üçgenin iç bölgesindedir..

Dik Üçgende Alan



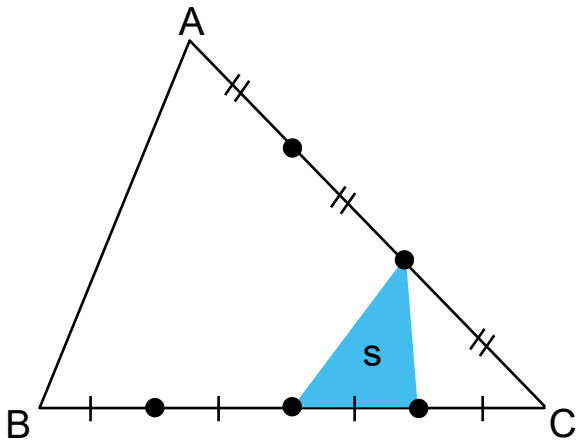
$$\text{Alan}(ABC) = \frac{x \cdot y}{2}$$

Dik üçgende alan kısa kenarların çarpımının yarısıdır.

TalesAR

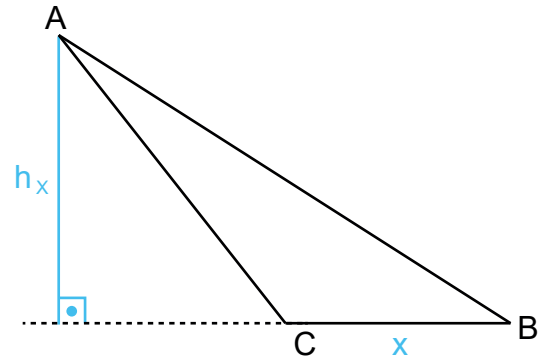


Ortak Yükseklik İle Alan Bölme



Toplamı kaç s olmalıdır?

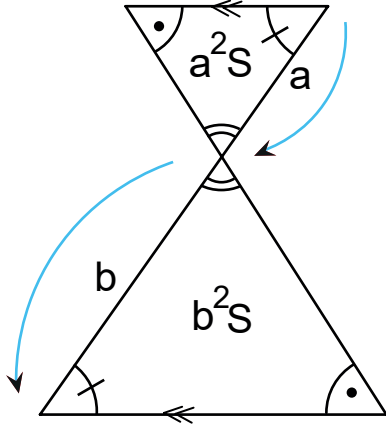
Geniş Açılı Üçgenlerde Alan



$$\text{Alan}(ABC) = \frac{x \cdot h_x}{2}$$

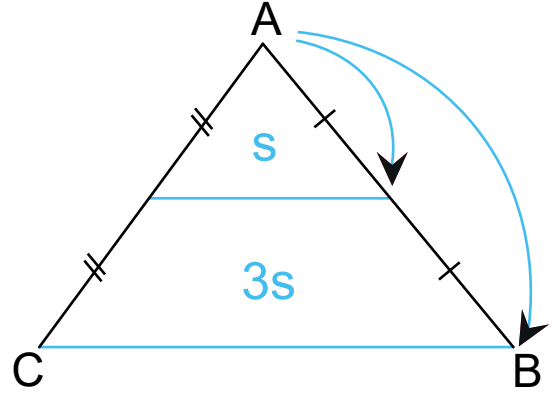
Dar açılı üçgenlerde diklik merkezi üçgenin iç bölgesindedir.

Kelebek Benzerliği Alan



$$\frac{a}{b} = \left(\frac{a}{b}\right)^2$$

Orta Tabanda Alan

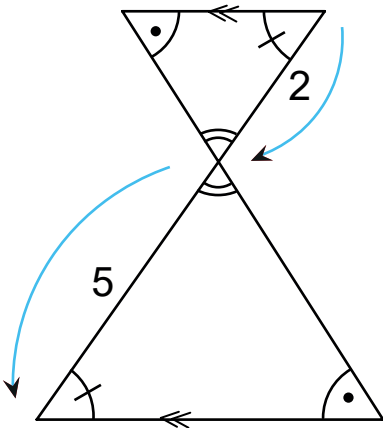


$$\text{Alansal Benzerlik} = \frac{1^2}{2^2}$$

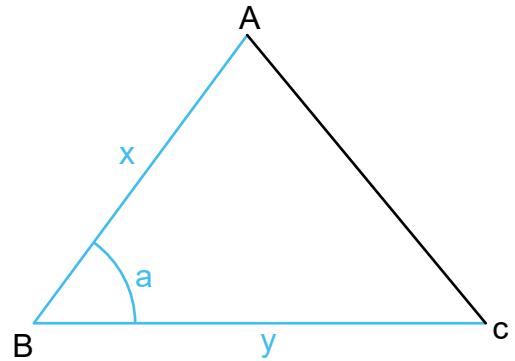
Alansal benzerlik oranı normal benzerlik oranının karesiyle doğru orantılıdır.

TalesAR

Örnek



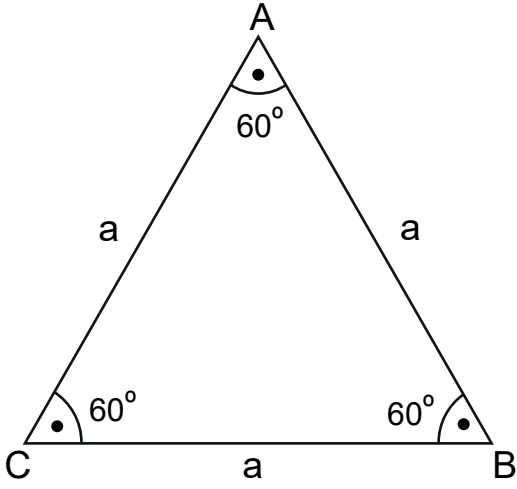
Sinüslü Alan



$$\text{Alan}(ABC) = \frac{x \cdot y}{2} \cdot \sin a$$

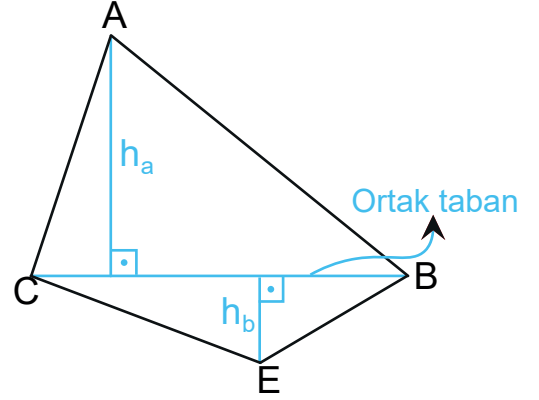


Eşkenar Üçgende Alan



$$\text{Alan}(ABC) = \frac{a^2 \cdot \sqrt{3}}{4}$$

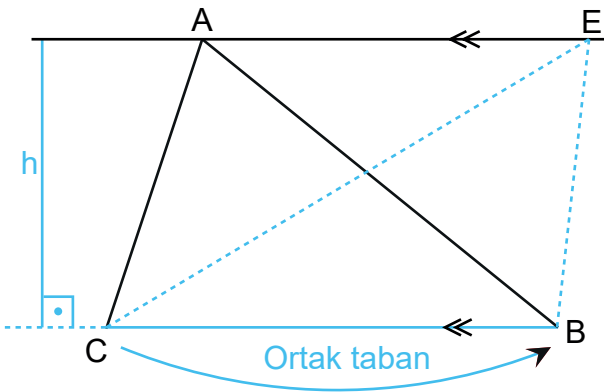
Ortak Tabanlı Üçgenlerde Alan



$$\frac{\text{Alan}(ABC) = \frac{h_a}{2} \cdot \text{Ortak taban}}{\text{Alan}(EBC) = \frac{h_b}{2} \cdot \text{Ortak taban}}$$

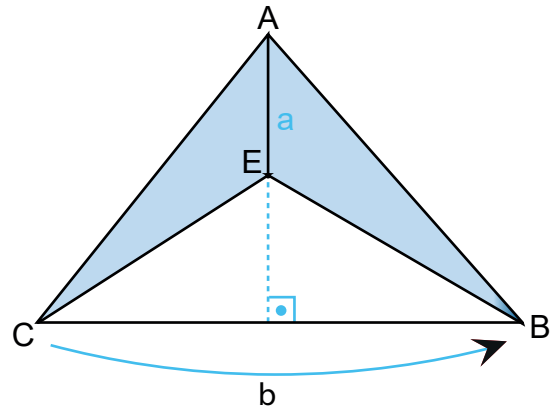
TalesAR

Ortak Yükseklik ve Tabanlı Üçgenler



$$\text{Alan}(ABC) = \text{Alan}(EBC)$$

Yükseklikler Toplamı

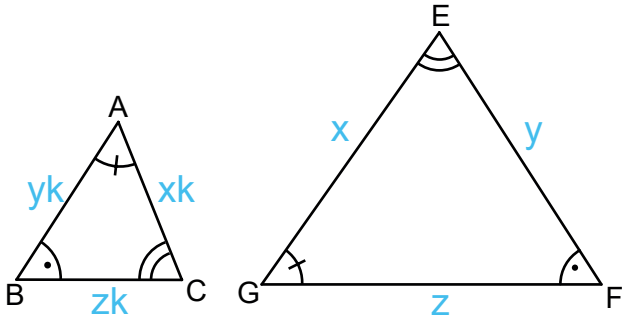


$$\text{Alan}(ABEC) = \frac{a \cdot b}{2}$$



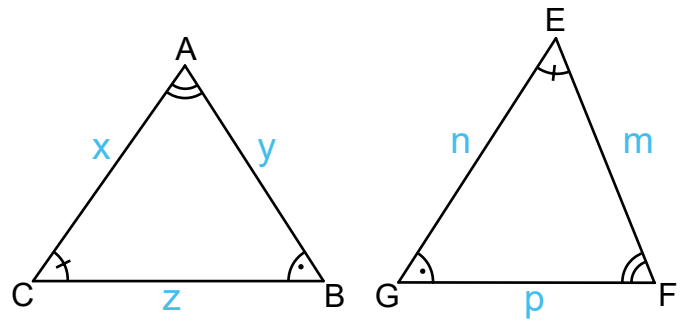
Benzerlik

Eşlik



$$\triangle ABC \sim \triangle EFG$$

$$\frac{xk}{x} = \frac{yk}{y} = \frac{zk}{z} = k$$



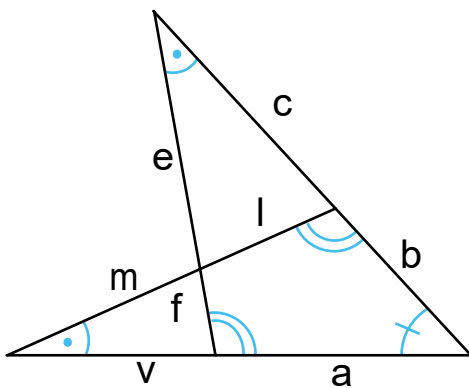
$$\triangle ABC \cong \triangle EFG$$

$$\frac{x}{m} = \frac{y}{p} = \frac{z}{n} = 1$$

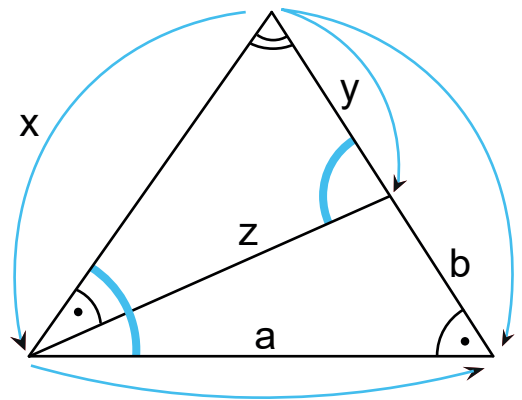
TalesAR

Bumerang Benzerliği

Büyük İçi Küçük Üçgen Benzerliği



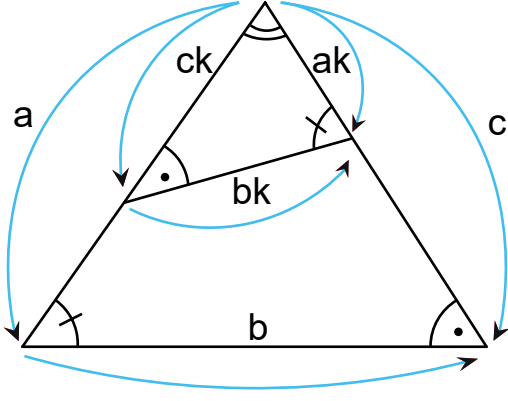
$$\frac{a}{b} = \frac{c+b}{v+a} = \frac{e+f}{m+l}$$



$$\frac{z}{a} = \frac{a}{y+b} = \frac{y}{x} = k$$

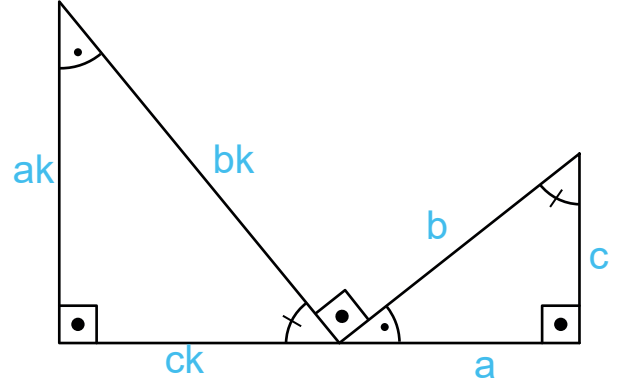


İç İçe Üçgen



$$\frac{ak}{a} = \frac{ck}{b} = \frac{bk}{c} = k$$

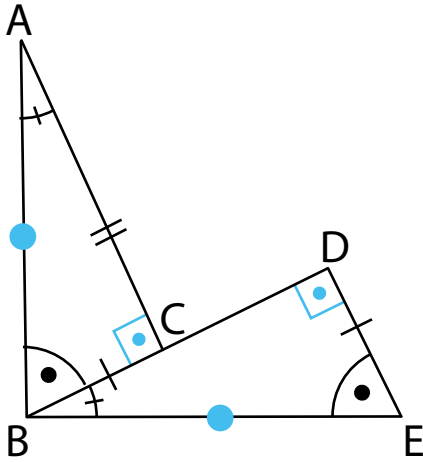
Dik Üçgen Benzerlik



$$\frac{ak}{a} = \frac{ck}{c} = \frac{bk}{b} = k$$

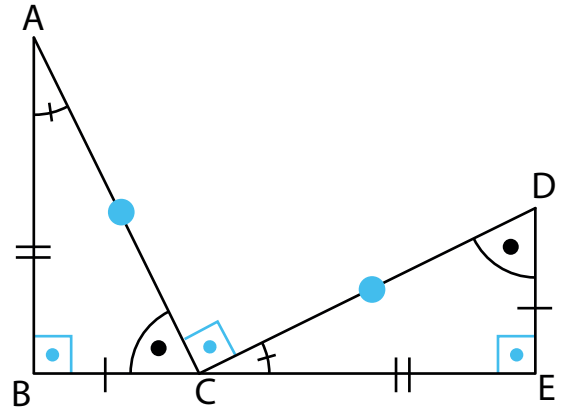
TalesAR

Kare Benzerliği



$$\begin{aligned} [AB] &= [BE] \\ [BC] &= [DE] \\ [AC] &= [BD] \end{aligned}$$

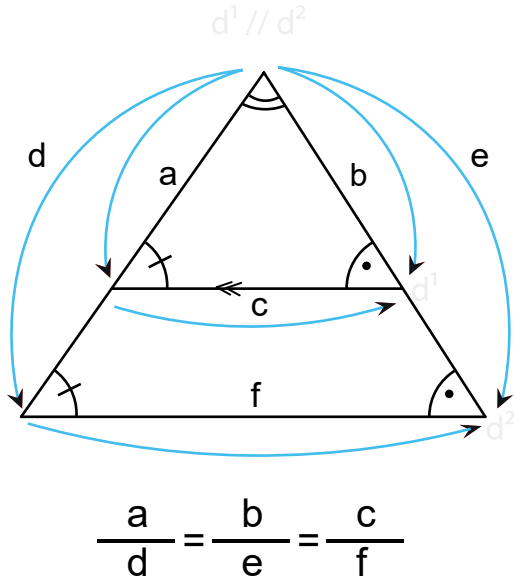
Dik Üçgen Eşlik



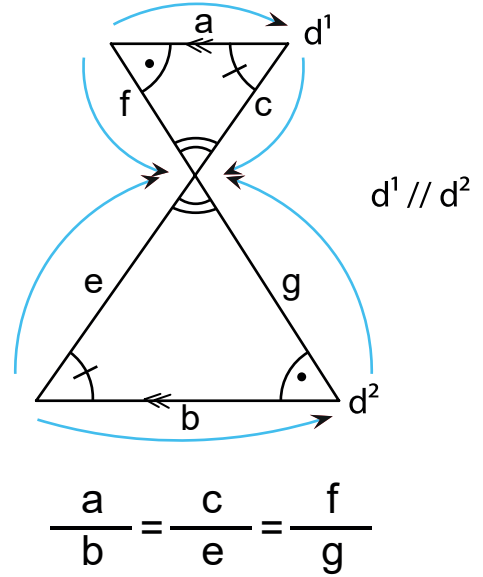
$$\begin{aligned} [AB] &= [CE] \\ [AC] &= [CD] \\ [BC] &= [DE] \end{aligned}$$



Katlanmış Kelebek

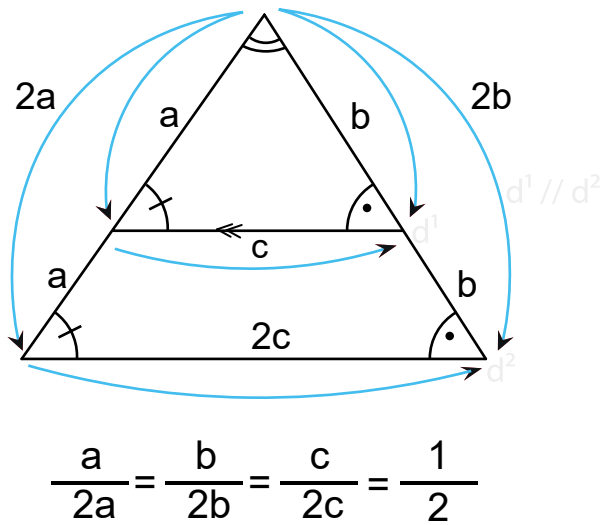


Kelebek Benzerliği

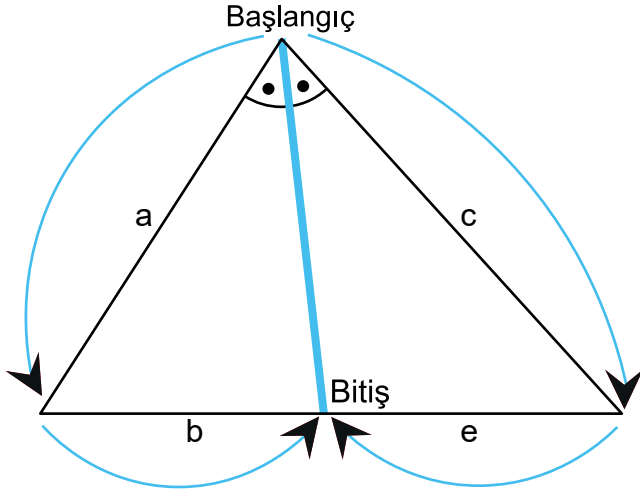


TalesAR

Orta Taban



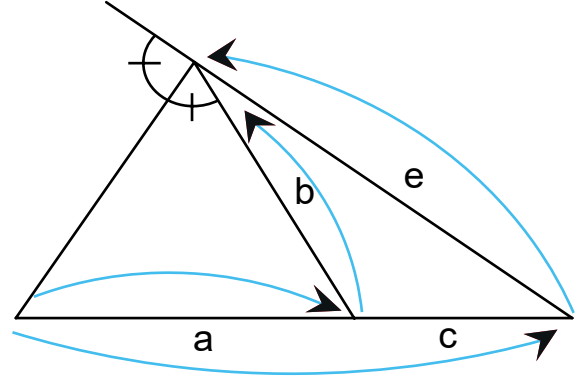
İç Açıortay Oranı



$$\frac{a}{b} = \frac{c}{e}$$

İç açıortay oranını yaparken açıortay doğruyu hiç dokunmuyoruz. Açıortay başlangıç noktasından çıkıp bitiş noktasına gitmeye çalışıyoruz ve bunları birbirine bölüyoruz.

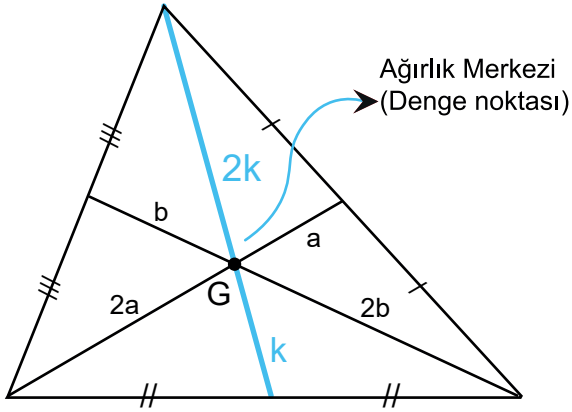
Dış Açıortay



$$\frac{a}{b} = \frac{a + c}{e}$$

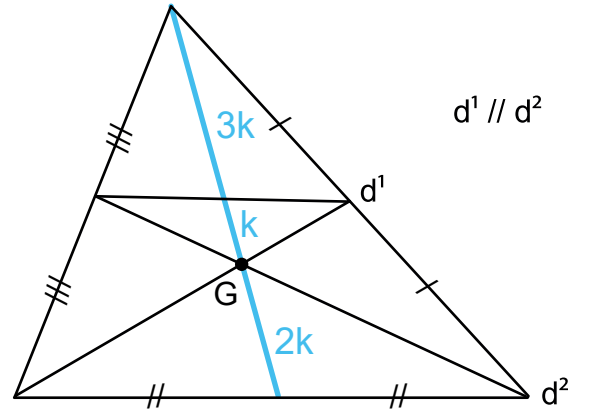
TalesAR

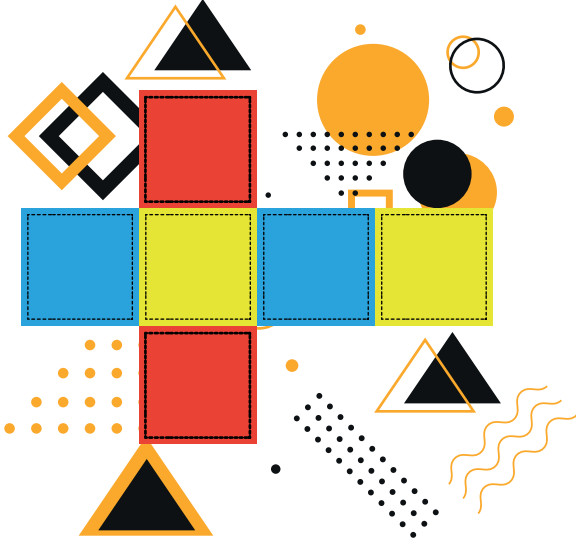
Kenarortay



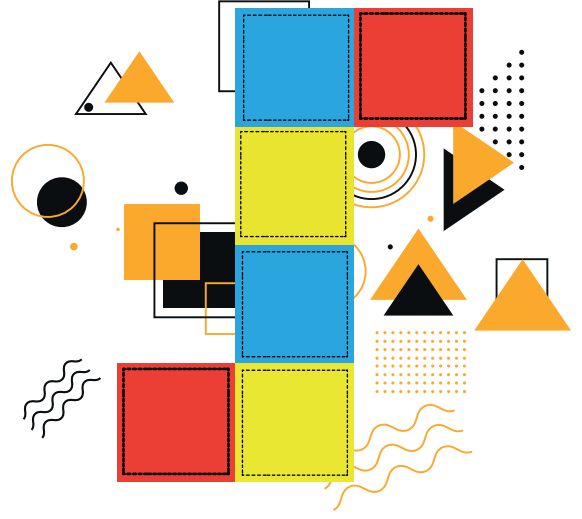
Kenarı ortalamayan doğruya kenarortay doğrusu denir. Üçgendeki üç kenarortayın kesim noktasına üçgenin ağırlık merkezi (denge noktası) denir. Köşeden ağırlık merkezine gelen doğru 2k, tabana inen doğru ise k olarak oran alır.

3 - 1 - 2 Kuralı



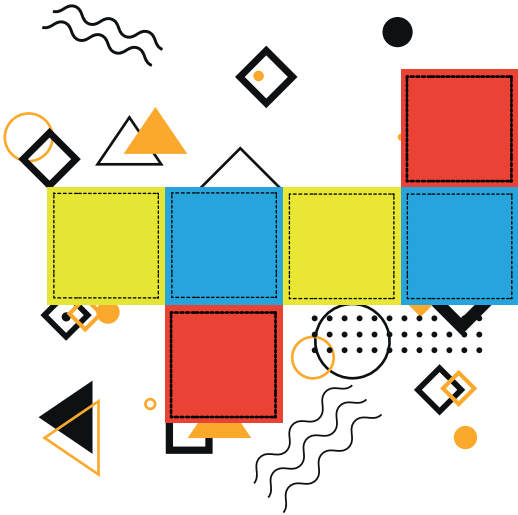


Küp
Yüzey alanı : $6a^2$

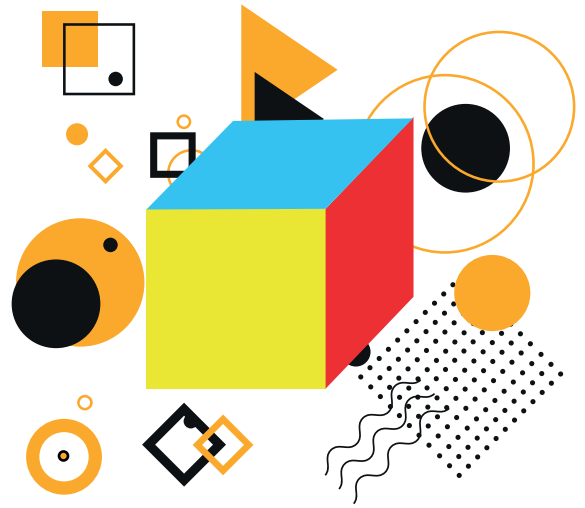


Küp Açılma

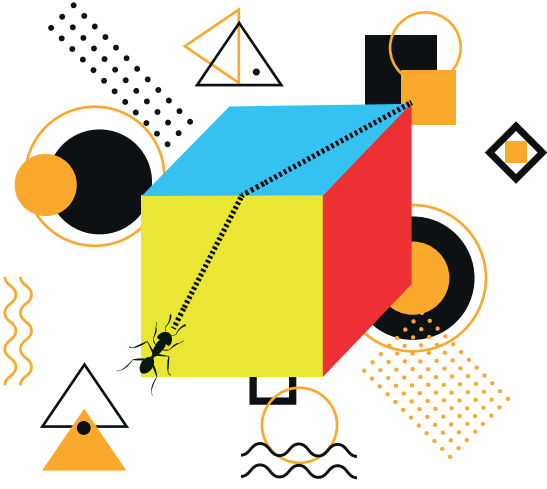
TalesAR



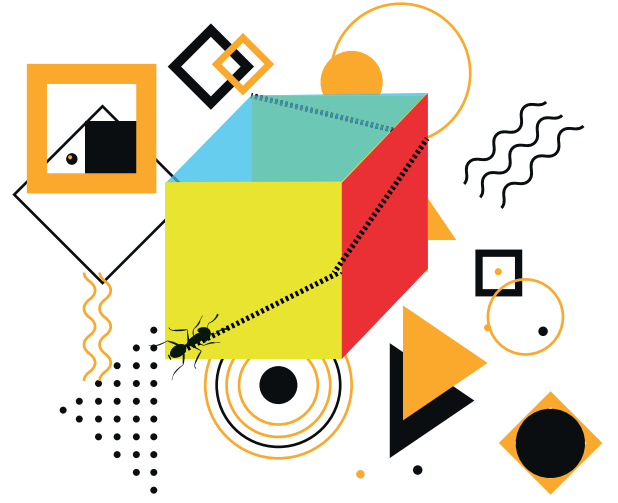
Küp Açılma



Küp
Yüzey alanı : $6a^2$
Hacim : a^3



Karınca Soruları
Yüzey açma

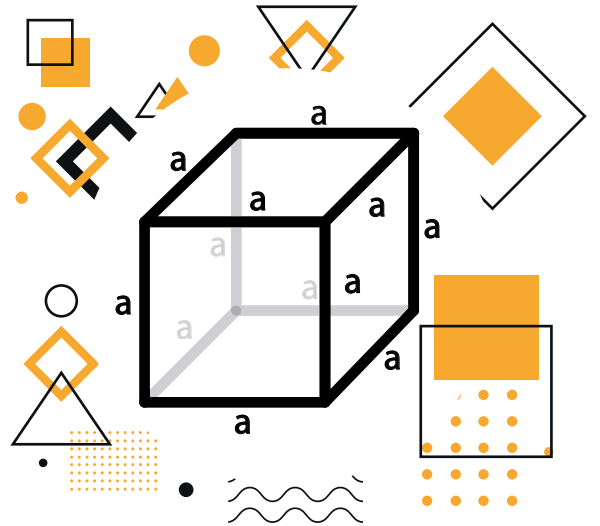


Karınca Soruları
Yüzey açma

TalesAR

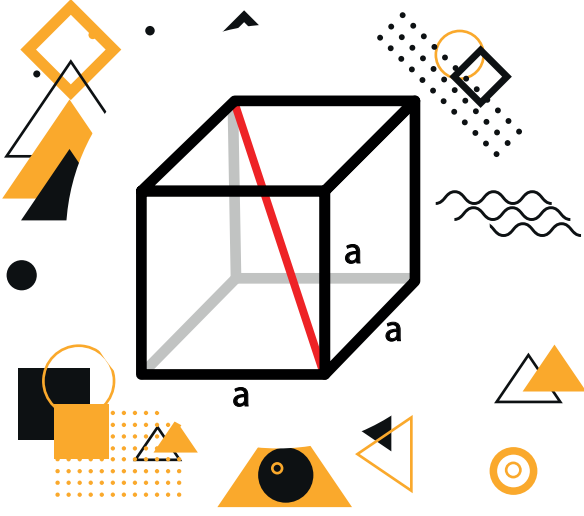


Yüzey Alanı
Değişmez

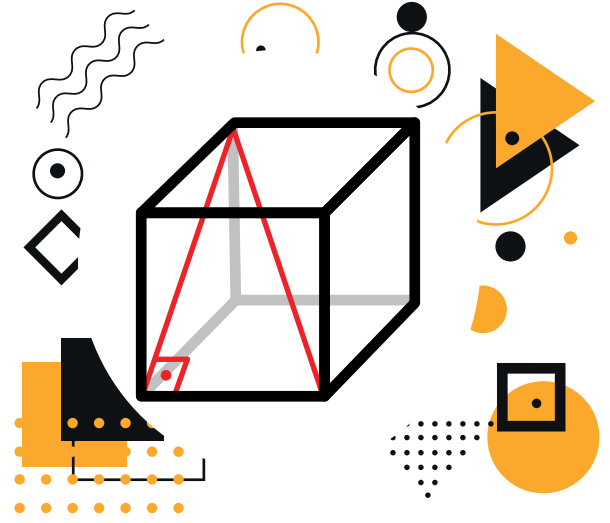


Küpün 12 ayrıtı vardır.

ŞİMDİ
DENE

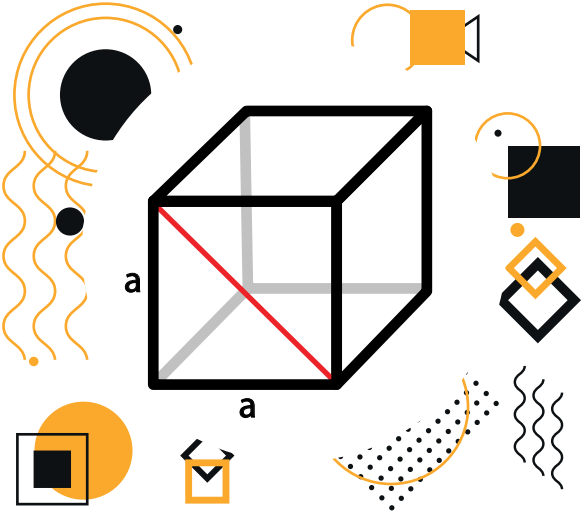


Küp Cisim Köşegeni : $a\sqrt{3}$

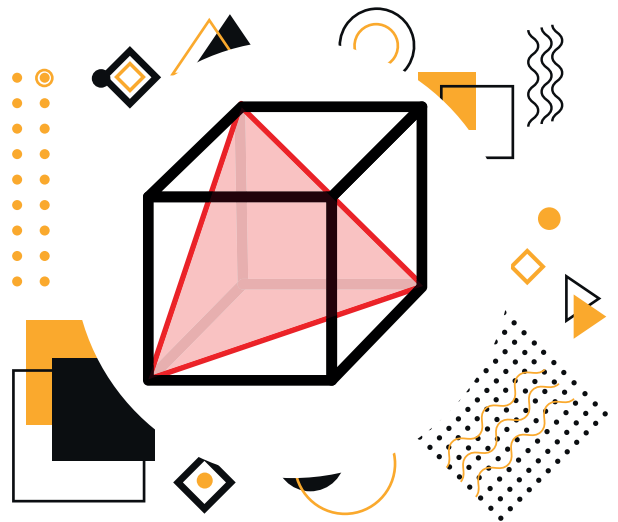


Küp İçi Dik Üçgen

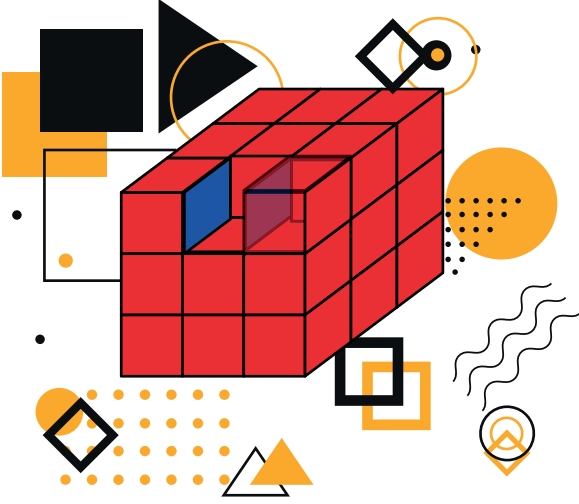
TalesAR



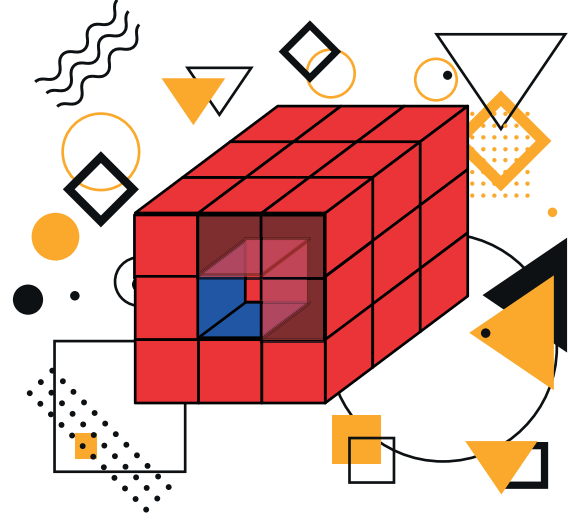
Küp Yüzey Köşegeni : $a\sqrt{2}$



Yüzey Köşegenleri
Eşkenar Üçgeni

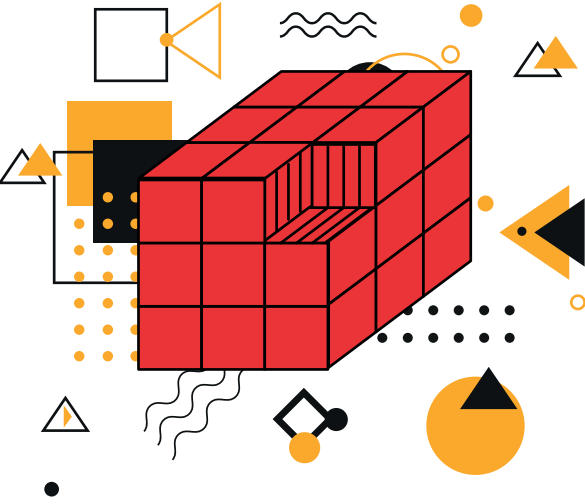


Kenardan Cisim Çıkarma :
İki Yüzey Artar.

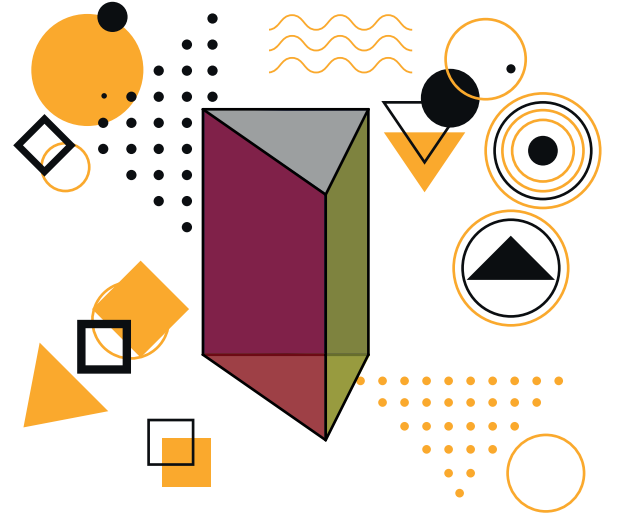


Ortadan Cisim Çıkarma :
Dört Yüzey Artar.

TalesAR



Köşeden Cisim Çıkarma :
Yüzey alanı değişmez.



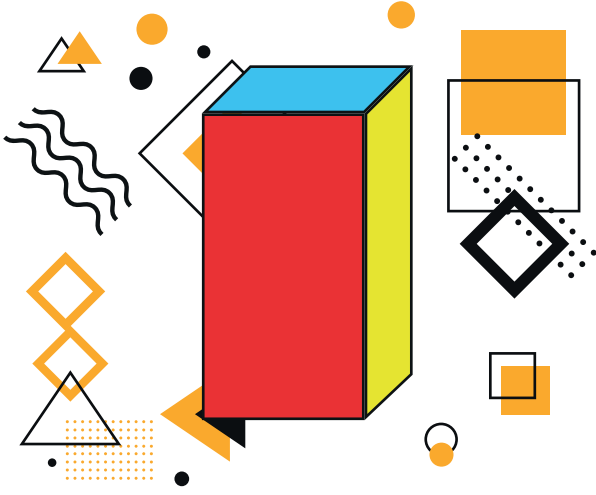
Üçgen Prizma

Yüzey alanı :

3 tane dikdörtgen + 2 üçgen

Hacim :

Üçgen Alanı x Yükseklik



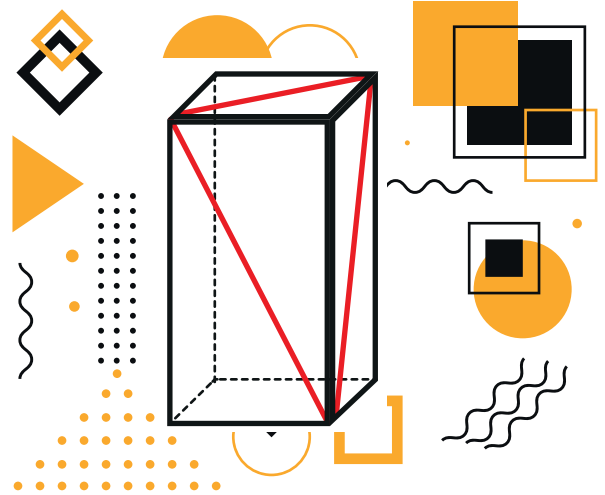
Dikdörtgenler Prizması

Yüzey alanı :

2 Sarı , 2 kırmızı , 2 mavi dikdörtgen

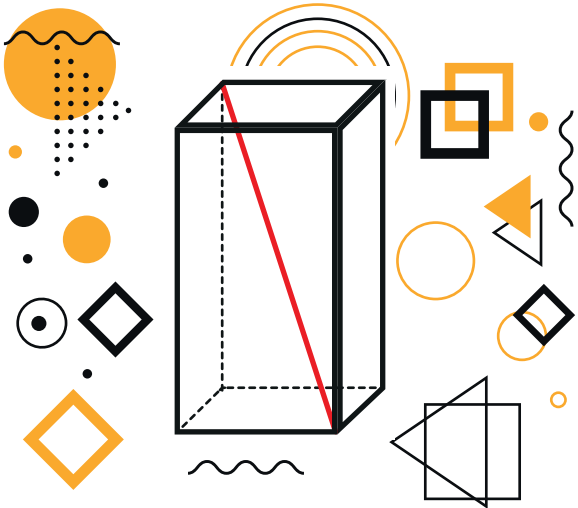
Hacim : Dikdörtgen Alanı x Yükseklik

TalesAR



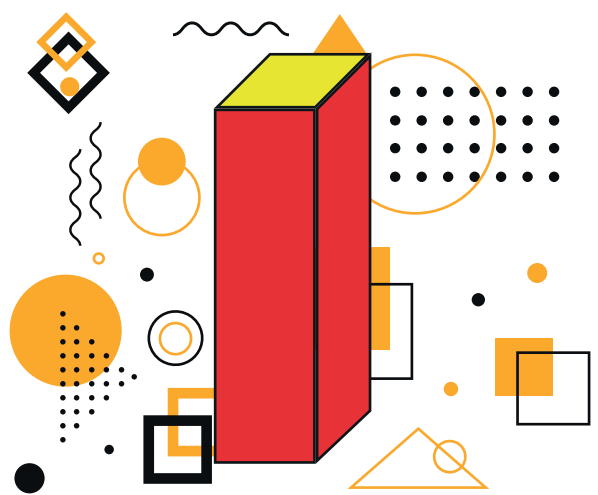
Dikdörtgenler Prizması

Yüzey Köşegenleri



Dikdörtgenler Prizması

Cisim Köşegeni



Kare Prizması

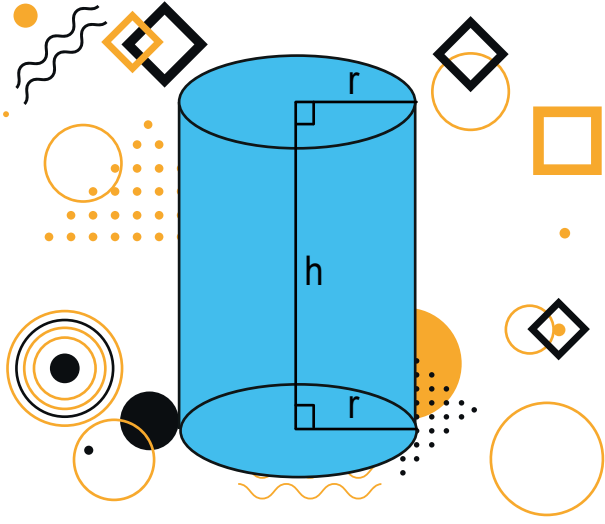
Yüzey alanı:

4 kırmızı eş dikdörtgen +

2 sarı kare

Hacim :

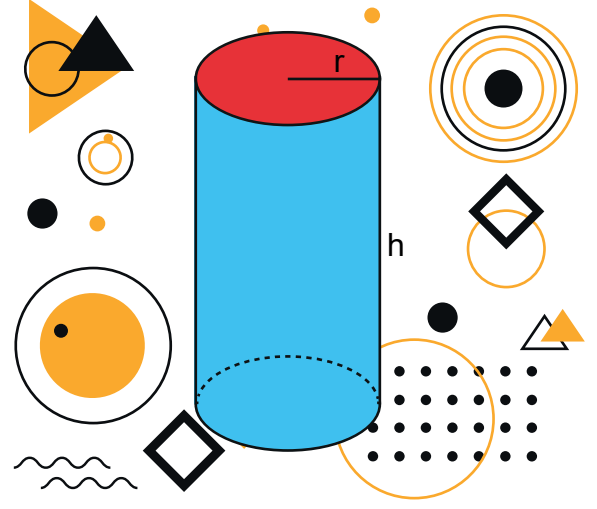
Karenin alanı x Yükseklik



Silindir

Hacim :

Çember Alanı x Yükseklik

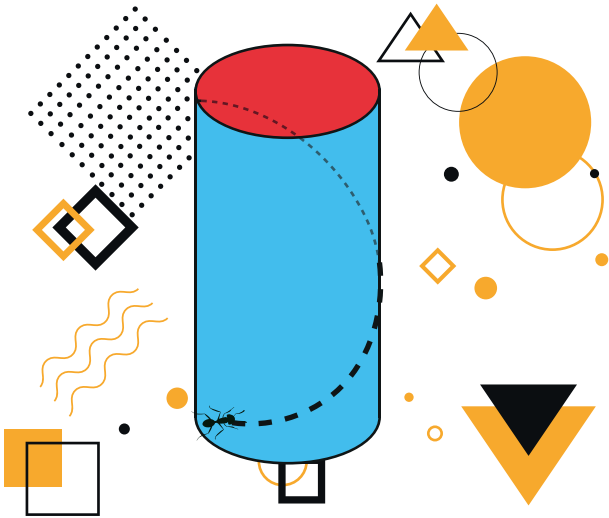


Silindir

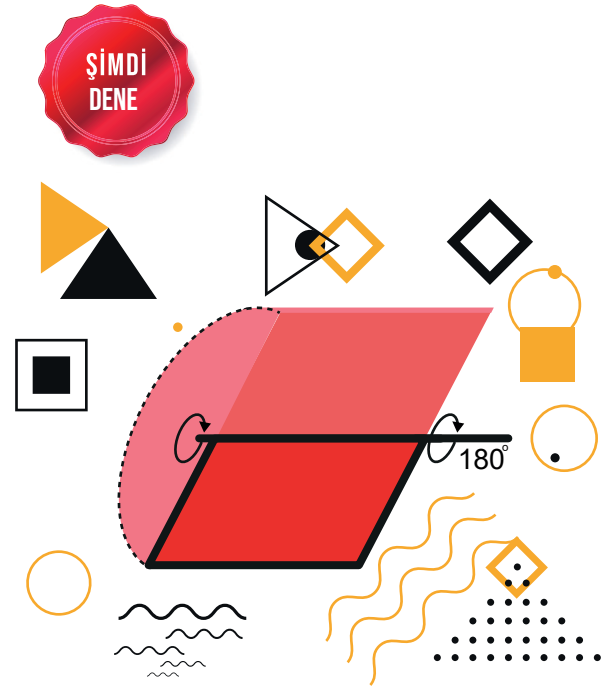
Yüzey alanı:

(Çember Çevre) x (Yükseklik) +
2 Kırmızı Çember Alanı

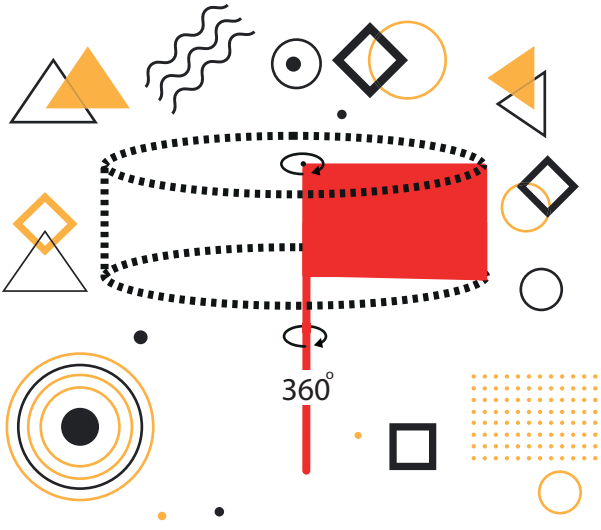
TalesAR



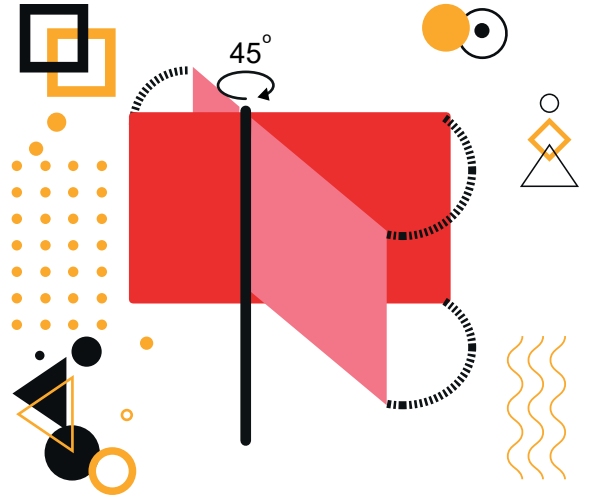
Silindir Karınca



Dikdörtgen Döndürmek
Silindir Oluşturur.

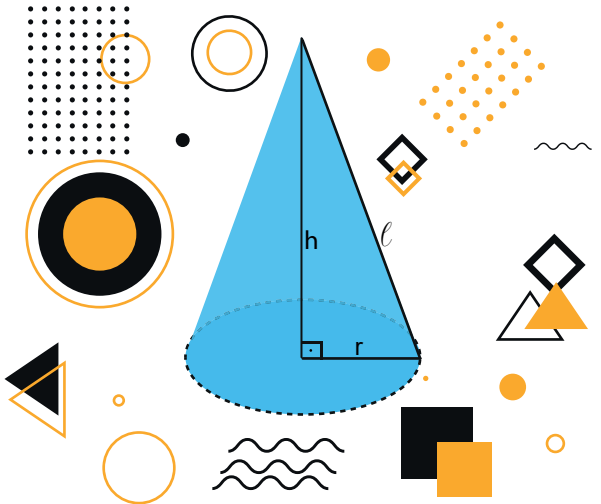


Dikdörtgen Döndürmek
Silindir Oluşturur.



Dikdörtgen Döndürmek
Silindir Oluşturur.

TalesAR

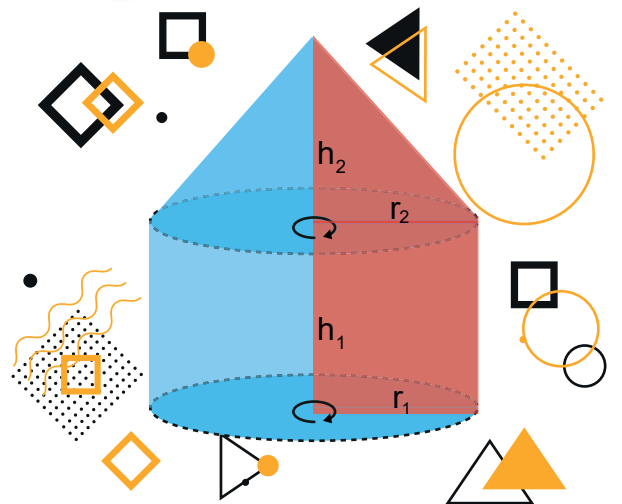


Koni

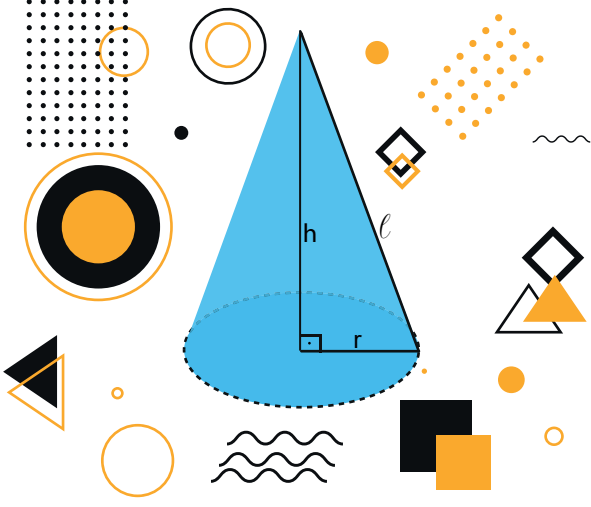
Hacim:

$(\text{Çember Alan}) \times (\text{Yükseklik})$

3



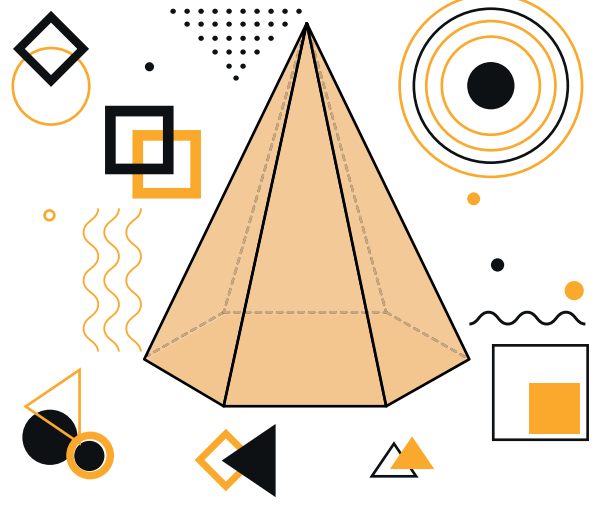
Yamuk Döndürmek
Silindir ve Koni Oluşturur.



Koni

Yüzey Alanı : $\pi r l$

$$\frac{r}{l} = \frac{\alpha}{360}$$



Altıgen Primit :

Hacim:

$$\frac{(\text{Altıgen}) \times (\text{Yükseklik})}{3}$$

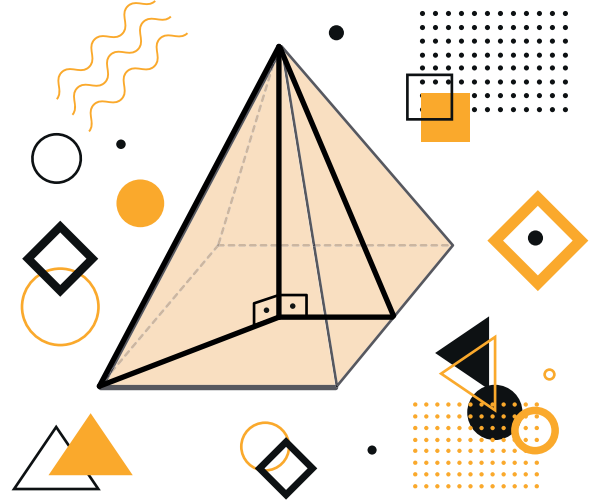
TalesAR



Kare Primit :

Hacim:

$$\frac{(\text{Kare}) \times (\text{Yükseklik})}{3}$$



Kare Primit

Yardımcı Üçgenler